WS 5.7 KEY

CALCULUS MAXIMUS



(8) (a) $f(x)=x^{2/3},\ c=8$

$f(8)=4$

$f'(x)=\frac{2}{3}x^{-1/3}=\frac{2}{3\sqrt[3]{x}}$

$f'(8)=\frac{1}{3}$

$L(x)=4+\frac{1}{3}(x-8)$

$f(8.06)\approx L(8.06)$

$L(8.06)=4+\frac{1}{3}(0.06)$

$=4+0.02$

$\boxed{L(8.06)=4.02}$

*over approximation because $f(x)$ is concave down $\forall x>0$.

** $f(8.06)=4.019975$

(b) $f(x)=\ln x,\ c=1$

$f(1)=0$

$f'(x)=\frac{1}{x},\ f'(1)=1$

$L(x)=0+1(x-1)$

$L(x)=x-1$

$f(1.07)\approx L(1.07)$

$L(1.07)=1.07-1$

$\boxed{L(1.07)=0.07}$

*over approximation because $f(x)$ is concave down $\forall x>0$

(9) Area $=\pi r^2=A$

$\frac{dA}{dr}=2\pi r$

$dA=2\pi r\,dr$

when $r=24,\ dr=0.2$

$dA=2\pi(24)(0.2)$

$\boxed{dA=9.6\pi\text{ cm}^2}$

(10) $f(1)=2,\ \frac{dy}{dx}=xy^3,\ \frac{d^2y}{dx^2}=y^3(1+3x^2y^2)$

(a) tangent line eq: $\boxed{y=2+8(x-1)}$

$\left.\frac{dy}{dx}\right|_{(1,2)}=8$

(b) $L(x)=2+8(x-1)$

$f(1.1)\approx L(1.1)=2+8(0.1)$

$\boxed{L(1.1)=2.8}$

$\frac{dy}{dx}=y^3(1+3x^2y^2)>0$ on $1<x<1.1$ because $y>0$ on $1<x<1.1$

so the given curve is concave up on $1<x<1.1$ and the tangent lines are <u>below</u> the curve of f on this interval SO $L(1.1)$ underapproximates $f(1.1)$

(11) (a) $f'(4)\approx\frac{f(5)-f(3)}{5-3}=\frac{-2-4}{2}=\boxed{-3}$

(b) $f'(5)=3,\ f(5)=-2$

*tangent line eq: $L(x)=-2+3(x-5)$

$f(7)\approx L(7)=-2+3(7-5)=4$

since $f''<0\ \forall x\in[5,8]$, $L(x)\geq f(x)$ on this interval, so $f(7)\geq L(7)=4$

*secant line eq: $f(8)=3,\ f(5)=-2$

slope $=\frac{-2-3}{5-8}=\frac{-5}{-3}=\frac{5}{3}$, using $f(8)=3$,

eq: $y(x)=3+\frac{5}{3}(x-8)$

$f(7)\approx y(7)=3+\frac{5}{3}(7-8)=3-\frac{5}{3}=\frac{4}{3}$

since $f''<0\ \forall x\in[5,8]$, $y(x)\leq f(x)$ on this interval, so $f(7)\geq y(7)=\frac{4}{3}$.

(12) $r(5)=30,\ r''<0$ for $0<t<12$

$r'(5)=2$

tangent line eq: $L(x)=30+2(x-5)$

$r(5.4)\approx L(5.4)=30+2(5.4-5)=\boxed{30.8\text{ ft}}$

Since $r''<0\ \forall t\in(0,12)$, r is concave down on this interval, so the tangent lines are above the graph of $r(x)$ on this interval, and so $L(5.4)$ <u>overapproximates</u> $r(5.4)$.