68). $4(\sin^6 x + \cos^6 x) = 4 - 3\sin^2 2x$ $(\sin 2x)(\sin 2x)$

RHS: $4 - 3(4\sin^2 x\cos^2 x) = 4 - 12\sin^2 x\cos^2 x$

LHS: $4\sin^6 x + 4\cos^6 x = 4\sin^2 x\sin^2 x\sin^2 x + 4\cos^2 x\cos^2 x\cos^2 x$

$= 4(1-\cos^2 x)^2\sin^2 x + 4(1-\sin^2 x)^2\cos^2 x = 4(1-\cos^4 x)\sin^2 x + 4(1-\sin^4 x$

$= 4(1-\cos^6 x) + 4(1-\sin^6 x)$

65) $$\frac{2(\tan x-\cot x)}{\tan^2 x-\cot^2 x}=\sin 2x$$

$a^2-b^2=(a+b)(a-b)$

LHS: $$\frac{2\left(\frac{\tan x}{1}-\frac{1}{\tan x}\right)}{\frac{\tan^2 x}{1}-\frac{1}{\tan^2 x}}=\frac{2\left(\frac{\tan^2 x-1}{\tan x}\right)}{\frac{\tan^4 x-1}{\tan^2 x}}=\frac{2\tan^2 x-1}{\tan x}\cdot\frac{\tan^2 x}{\tan^4 x-1}$$

$$=\frac{2(\tan^2 x-1)}{\tan x}\cdot\frac{\tan^2 x}{(\tan^2 x+1)(\tan^2 x-1)}=\frac{2\tan x}{\sec^2 x}=2\,\frac{\sin x}{\cos x}\cdot\frac{\cos^2 x}{1}$$

$$=2\sin x\cos x$$

$$=\sin 2x$$

66) $\cot 2x = \frac{1-\tan^2 x}{2\tan x}$

LHS: $\frac{1}{\tan 2x} = \frac{1}{\frac{2\tan x}{1-\tan^2 x}} = \text{RHS}$.

63) $\frac{\sin 4x}{\sin x} = 4\cos x \cos 2x$

LHS: $\frac{2\sin 2x \cos 2x}{\sin x} = \frac{2(2\sin x \cos x)\cos 2x}{\sin x}$

$= 4\cos x \cos 2x = RHS$ ✓

61) $(\sin x + \cos x)^2 = 1 + \sin 2x$

$a^2 - b^2 = (a+b)(a-b)$

LHS $= \sin^2 x + 2\sin x\cos x + \cos^2 x = 1 + 2\sin x\cos x = 1 + \sin 2x = RHS$

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65) $\dfrac{2(\tan x - \cot x)}{\tan^2 x - \cot^2 x} = \sin 2x$

LHS: $\dfrac{2(\tan x - \cot x)}{(\tan x + \cot x)(\tan x - \cot x)} = \dfrac{2}{\tan x + \cot x} = \dfrac{2}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}}$

$= \dfrac{2}{\frac{(\sin^2 x + \cos^2 x)}{\sin x \cos x}} = 2\sin x\cos x = \sin 2x = RHS$ ✓

61) $(\sin x + \cos x)^2 = 1 + \sin 2x$

LHS: $\sin^2 x + 2\sin x\cos x + \cos^2 x = 1 + 2\sin x\cos x$

$= 1 + \sin 2x = RHS$ ✓

63) $\dfrac{\sin 4x}{\sin x} = 4\cos x \cos 2x$

LHS: $\dfrac{2\sin 2x \cos 2x}{\sin x} = \dfrac{2(2\sin x \cos x)\cos 2x}{\sin x}$

$= 4\cos x \cos 2x = \text{RHS}$ ✓

21) $\sin^2\frac{x}{2} = \frac{1-\cos x}{2}$

LHS: $\left(\pm\sqrt{\frac{1-\cos x}{2}}\right)^2$

$= \frac{1-\cos x}{2} = RHS$ ✓

23) $\cot\frac{\theta}{2} = \frac{\sin\theta}{1-\cos\theta}$

LHS: $\frac{1}{\tan\frac{\theta}{2}} = \frac{1}{\frac{1-\cos\theta}{\sin\theta}}$

$= \frac{\sin\theta}{1-\cos\theta} = RHS$ ✓

20) $1 + \sin 2t = (\sin t + \cos t)^2$

LHS $= 1 + 2\sin t\cos t$

RHS: $\sin^2 t + 2\sin t\cos t + \cos^2 t$

$= 1 + 2\sin t\cos t$

25) $\cos 2u = \dfrac{1-\tan^2 u}{1+\tan^2 u}$

RHS: $\dfrac{1-\tan^2 u}{\sec^2 u} = \dfrac{1-\dfrac{\sin^2 u}{\cos^2 u}}{\dfrac{1}{\cos^2 u}} = \dfrac{\cos^2 u - \sin^2 u}{\cancel{\cos^2 u}} \cdot \dfrac{\cancel{\cos^2 u}}{1}$

$= \cos 2u$

$= \text{RHS}$ ✓

26) $$\frac{\cos 2u}{1-\sin 2u} = \frac{1+\tan u}{1-\tan u}$$

$$\frac{a+b}{a-b}$$

RHS: $$\frac{1+\frac{\sin u}{\cos u}}{1-\frac{\sin u}{\cos u}} = \frac{\frac{\cos u+\sin u}{\cos u}}{\frac{\cos u-\sin u}{\cos u}} = \frac{\cos u+\sin u}{\cancel{\cos u}} \cdot \frac{\cancel{\cos u}}{\cos u-\sin u}$$

$$= \frac{\cos u+\sin u}{\cos u-\sin u} \cdot \frac{\cos u-\sin u}{\cos u-\sin u} = \frac{\cos^2 u-\sin^2 u}{\cos^2 u-2\cos u\sin u+\sin^2 u}$$

$$= \frac{\cos 2u}{1-\sin 2u} = \text{RHS}$$

~~$\frac{1+\sin 2u}{\cos 2u}$~~

28) $\sec 2x = \dfrac{\sec^2 x}{2 - \sec^2 x}$

RHS: $\dfrac{1+\tan^2 x}{2-(1+\tan^2 x)} = \dfrac{1+\tan^2 x}{2-1-\tan^2 x} = \dfrac{1+\tan^2 x}{1-\tan^2 x} = \dfrac{1}{\dfrac{\cos^2 x - \sin^2 x}{\cos^2 x}}$

[illegible] $\dfrac{1}{\cos^2 x - \sin^2 x} = \dfrac{1}{\cos 2x} = \sec 2x$ = LHS ✓

22) $\cos^2\frac{x}{2} = \frac{1+\cos x}{2}$

LHS: $\left(\pm\sqrt{\frac{1+\cos x}{2}}\right)^2 = \frac{1+\cos x}{2} = \text{RHS}$ ✓

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23) $\cot\frac{\theta}{2} = \frac{\sin\theta}{1-\cos\theta}$

LHS: $\frac{1}{\tan\frac{\theta}{2}} = \frac{1}{\frac{1-\cos\theta}{\sin\theta}} = \frac{\sin\theta}{1-\cos\theta} = \text{RHS}$ ✓

25) $\cos 2u = \dfrac{1-\tan^2 u}{1+\tan^2 u}$

RHS: $\dfrac{1-\tan^2 u}{\sec^2 u} = \dfrac{1 - \dfrac{\sin^2 u}{\cos^2 u}}{\dfrac{1}{\cos^2 u}} = \dfrac{\dfrac{\cos^2 u - \sin^2 u}{\cos^2 u}}{\dfrac{1}{\cos^2 u}}$

$= \dfrac{\cos^2 u - \sin^2 u}{\cancel{\cos^2 u}} \cdot \dfrac{\cancel{\cos^2 u}}{1} = \cos 2u = \text{LHS}$ ✓

28) $\sec 2x = \dfrac{\sec^2 x}{2-\sec^2 x}$

LHS: $\dfrac{1}{\cos 2x} = \dfrac{1}{\cos^2 x - \sin^2 x}$

RHS: $\dfrac{\frac{1}{\cos^2 x}}{2 - \frac{1}{\cos^2 x}} = \dfrac{\frac{1}{\cos^2 x}}{\frac{2\cos^2 x - 1}{\cos^2 x}} = \dfrac{1}{\cancel{\cos^2 x}} \cdot \dfrac{\cancel{\cos^2 x}}{2\cos^2 x - 1}$

$= \dfrac{1}{2\cos^2 x - 1}$

$= \dfrac{1}{\cos 2x}$

$= \sec 2x$ LHS

26) $\frac{\cos 2u}{1-\sin 2u} = \frac{1+\tan u}{1-\tan u}$

$\frac{a+b}{a-b}$

LHS: $\frac{\cos^2 u - \sin^2 u}{1 - 2\sin u \cos u}$

RHS: $\frac{1+\tan u}{1-\tan u} \cdot \frac{1-\tan u}{1-\tan u} = \frac{1-\tan^2 u}{1-2\tan u+\tan^2 u} = \frac{1-\tan^2 u}{\sec^2 u - 2\tan u}$

$\frac{\sin^2 u}{\cos^2 u}$, $\frac{1}{\cos^2}$

$= \frac{\frac{\cos^2 u - \sin^2 u}{\cos^2 u}}{\frac{1-2\sin u\cos u}{\cos^2 u}} = \frac{\cos^2 u - \sin^2 u}{\cancel{\cos^2 u}} \cdot \frac{\cancel{\cos^2 u}}{1-2\sin u\cos u} = \frac{\cos 2u}{1-\sin 2u}$

$=$ LHS

26) $\frac{\cos 2u}{1-\sin 2u} = \frac{1+\tan u}{1-\tan u}$

$\frac{a+b}{a-b}$

RHS: $\frac{1+\frac{\sin u}{\cos u}}{1-\frac{\sin u}{\cos u}} = \frac{\frac{\cos u+\sin u}{\cos u}}{\frac{\cos u-\sin u}{\cos u}} = \frac{\cos u+\sin u}{\cancel{\cos u}} \cdot \frac{\cancel{\cos u}}{\cos u-\sin u}$

$= \frac{\cos u+\sin u}{\cos u-\sin u} \cdot \frac{\cos u-\sin u}{\cos u-\sin u} = \frac{\cos^2 u-\sin^2 u}{\cos^2 u-2\cos u\sin u+\sin^2 u}$

$= \frac{\cos 2u}{1-2\cos u\sin u} = \frac{\cos 2u}{1-\sin 2u}$

$= LHS$

28) $\sec 2x = \dfrac{\sec^2 x}{2-\sec^2 x}$

RHS: $\dfrac{\frac{1}{\cos^2 x}}{2-\frac{1}{\cos^2 x}} = \dfrac{\frac{1}{\cos^2 x}}{\frac{2\cos^2 x - 1}{\cos^2 x}} = \dfrac{1}{2\cos^2 x - 1}$

$= \dfrac{1}{\cos 2x}$

$= \sec 2x$

$=$ LHS ✓