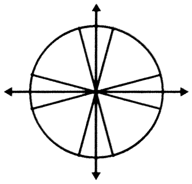

$\cos 15^\circ = \dfrac{\sqrt{6}+\sqrt{2}}{4}$

$\sin 15^\circ = \dfrac{\sqrt{6}-\sqrt{2}}{4}$

Trigonometry

LESSON SEVEN - *Trigonometric Identities II*

Lesson Notes

Example 9 Prove each trigonometric identity.

Double-Angle Identities

$\sin 2x = 2\sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = 2\cos^2 x - 1$

$\cos 2x = 1 - 2\sin^2 x$

$\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

a) $\cos 2x + 2\sin^2 x = 1$

LHS: $\cos^2 x - \sin^2 x + 2\sin^2 x$

$= \cos^2 x + \sin^2 x = 1 =$ RHS ✓

b) $\dfrac{2}{1+\cos 2x} = \sec^2 x$

LHS: $\dfrac{2}{1+2\cos^2 x - \cancel{1}} = \dfrac{\cancel{2}}{\cancel{2}\cos^2 x}$

$\Rightarrow \dfrac{1}{\cos^2 x} = \sec^2 x$

$=$ RHS ✓

c) $\dfrac{\sin 2x}{\cos 2x + \sin^2 x} = 2\tan x$

LHS: $\dfrac{2\sin x \cos x}{\cos^2 x - \cancel{\sin^2 x} + \cancel{\sin^2 x}} = \dfrac{2\sin x \cancel{\cos x}}{\cos^{\cancel{2}} x}$

$\Rightarrow 2\dfrac{\sin x}{\cos x} = 2\tan x =$ RHS ✓

d) $\dfrac{2\sin x \cos x}{\cos^2 x - \sin^2 x} = \tan 2x$

LHS: $\dfrac{\sin 2x}{\cos 2x} = \tan 2x =$ RHS ✓

Trigonometry
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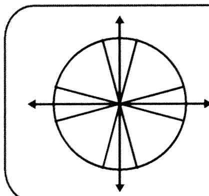


$\cos 15^\circ = \frac{\sqrt{6}+\sqrt{2}}{4}$

$\sin 15^\circ = \frac{\sqrt{6}-\sqrt{2}}{4}$

Example 10

Prove each trigonometric identity.

Double-Angle Identities

$\sin 2x = 2\sin x\cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = 2\cos^2 x - 1$

$\cos 2x = 1 - 2\sin^2 x$

$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

a) $\cos^4 x - \sin^4 x = \cos 2x$

LHS: $(\cos^2 x - \sin^2 x)(\cancel{\cos^2 x + \sin^2 x})$

$\Rightarrow \cos^2 x - \sin^2 x = \cos 2x = RHS$

b) $1-(\sin x + \cos x)^2 = -\sin 2x$

LHS: $1-(\sin^2 x + 2\sin x\cos x + \cos^2 x)$

$= 1-(1+2\sin x\cos x)$

$= 1-1-2\sin x\cos x$

$= -2\sin x\cos x = -\sin 2x$

$= RHS$

c) $\frac{2(\tan x - \cot x)}{\tan^2 x - \cot^2 x} = \sin 2x$

LHS: $\frac{2(\cancel{\tan x - \cot x})}{(\tan x + \cot x)(\cancel{\tan x - \cot x})}$

$= \frac{2}{\tan x + \cot x} = \frac{2}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}}$

$= \frac{2}{\frac{\sin^2 x + \cos^2 x}{\sin x\cos x}} = \frac{2}{1}\cdot\frac{\sin x\cos x}{\cancel{\sin^2 x + \cos^2 x}}$

$= 2\sin x\cos x = \sin 2x$

$= RHS$

d) $\frac{1}{1-\tan x} - \frac{1}{1+\tan x} = \tan 2x$

LHS: $\frac{1+\tan x - (1-\tan x)}{(1-\tan x)(1+\tan x)} = \frac{1+\tan x - 1 + \tan x}{(1-\tan x)(1+\tan x)}$

$= \frac{2\tan x}{(1-\tan x)(1+\tan x)} = \frac{2\tan x}{1-\tan^2 x} = \tan 2x$

$= RHS$

Example 11 Prove each trigonometric identity. **Assorted Proofs**

a) $2\csc 2x = \csc x \sec x$

RHS: $\dfrac{1}{\sin x}\cdot\dfrac{1}{\cos x} = \dfrac{2\cdot 1}{2\cdot \sin x \cos x}$

$= \dfrac{2}{2\sin x \cos x} = \dfrac{2}{\sin 2x} = 2\csc 2x$

$=$ LHS

b) $\dfrac{\sin(x+y)}{\cos x \sin y} = \tan x \cot y + 1$

LHS: $\dfrac{\sin x \cos y + \cos x \sin y}{\cos x \sin y}$

$= \dfrac{\sin x \cos y}{\cos x \sin y} + \dfrac{\cancel{\cos x}\,\cancel{\sin y}}{\cancel{\cos x}\,\cancel{\sin y}}$

$= \tan x \cot y + 1 =$ RHS

c) $\sin 88^\circ \cos 58^\circ - \cos 88^\circ \sin 58^\circ = \dfrac{1}{2}$

d) $\tan\left(x+\dfrac{\pi}{4}\right) = \dfrac{\tan x + 1}{1-\tan x}$

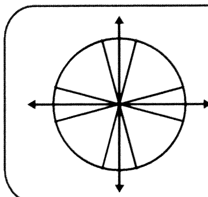
LHS: $\dfrac{\tan x + \tan \pi/4}{1-\tan x \tan \pi/4} = \dfrac{\tan x + 1}{1-\tan x\,(1)} = \dfrac{\tan x + 1}{1-\tan x}$

$=$ RHS

Trigonometry

LESSON SEVEN- *Trigonometric Identities II*

Lesson Notes



$\cos 15° = \dfrac{\sqrt{6}+\sqrt{2}}{4}$

$\sin 15° = \dfrac{\sqrt{6}-\sqrt{2}}{4}$

Example 12 Prove each trigonometric identity. **Assorted Proofs**

a) $(\sin x + \cos x)^2 - 1 = \sin 2x$

LHS: $\sin^2 x + 2\sin x\cos x + \cos^2 x - 1$

$= \cancel{1} + 2\sin x\cos x \cancel{- 1} = 2\sin x\cos x$

$= \sin 2x$

$= RHS$

b) $\dfrac{1}{2}\sin\dfrac{2x}{5} = \sin\dfrac{x}{5}\cos\dfrac{x}{5}$

$\frac{1}{2}\sin\left(2\left(\frac{x}{5}\right)\right)$

LHS: $\frac{1}{\cancel{2}}\left(\cancel{2}\sin\frac{x}{5}\cos\frac{x}{5}\right) = \sin\frac{x}{5}\cos\frac{x}{5} = RHS$

c) $\cos^2\left(x - \dfrac{\pi}{2}\right) = \sin^2 x$

LHS: $\cancel{\cos^2 x \overset{0}{\cos^2\left(\frac{\pi}{2}\right)}} + \sin^2 x \overset{1}{\sin^2\left(\frac{\pi}{2}\right)}$

$= \sin^2 x = RHS$

d) $\sin 3x = 3\sin x - 4\sin^3 x$

LHS: $\sin(2x + x) = \sin 2x\cos x + \cos 2x\sin x$

$= 2\cos x\sin x\cos x + (\cos^2 x - \sin^2 x)\sin x$

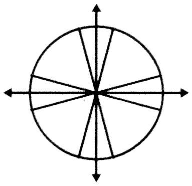
$= 2\cos^2 x\sin x + \sin x\cos^2 x - \sin^3 x$

$= 3\cos^2 x\sin x - \sin^3 x$

$= 3(1 - \sin^2 x)\sin x - \sin^3 x = (3 - 3\sin^2 x)\sin x - \sin^3 x$

$= 3\sin x - 3\sin^3 x - \sin^3 x = 3\sin x - 4\sin^3 x$

$= RHS$



$\cos 15^\circ = \dfrac{\sqrt{6}+\sqrt{2}}{4}$

$\sin 15^\circ = \dfrac{\sqrt{6}-\sqrt{2}}{4}$

Trigonometry

LESSON SEVEN - *Trigonometric Identities II*

Lesson Notes

Example 13 Prove each trigonometric identity.

Assorted Proofs

a) $\dfrac{5\sin x - \cos 2x - 11}{2\sin x - 3} = \sin x + 4$

$$LS = \frac{5\sin x - (1 - 2\sin^2 x) - 11}{2\sin x - 3} = \frac{5\sin x - 1 + 2\sin^2 x - 11}{2\sin x - 3}$$

$$= \frac{5\sin x + 2\sin^2 x - 12}{2\sin x - 3} = \frac{2\sin^2 x + 5\sin x - 12}{2\sin x - 3}$$

$$\frac{\cancel{(2\sin x - 3)}(\sin x + 4)}{\cancel{2\sin x - 3}} = \sin x + 4 = RHS$$

b) $\cos 3x = 4\cos^3 x - 3\cos x$

$$LHS: \cos(2x + x) = \cos 2x \cos x - \sin 2x \sin x$$

$$= (2\cos^2 x - 1)\cos x - 2\sin x \cos x \sin x$$

$$= 2\cos^3 x - \cos x - 2\sin^2 x \cos x$$

$$= 2\cos^3 x - \cos x - 2(1 - \cos^2 x)\cos x$$

$$= 2\cos^3 x - \cos x + (-2 + 2\cos^2 x)\cos x$$

$$= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x$$

$$= 4\cos^3 x - 3\cos x = RHS$$

c) $\cos 34^\circ \cos 41^\circ - \sin 34^\circ \sin 41^\circ = \dfrac{\sqrt{6}-\sqrt{2}}{4}$

d) $\dfrac{\tan x + \tan y}{\sec x \sec y} = \sin(x + y)$

$$LHS = \frac{\dfrac{\sin x}{\cos x} + \dfrac{\sin y}{\cos y}}{\sec x \sec y} = \frac{\dfrac{\sin x \cos y + \sin y \cos x}{\cos x \cos y}}{\sec x \sec y}$$

$$= \frac{\sin x \cos y + \sin y \cos x}{\cos x \cos y} \cdot \frac{1}{\sec x \sec y}$$

$$= \frac{\sin x \cos y + \sin y \cos x}{\cancel{\cos x \cos y}} \cdot \cancel{\cos x \cos y}$$

$$= \sin x \cos y + \sin y \cos x = \sin(x + y) = RHS$$