Word Problems Using Functions

If the speed limit on a 100-mile stretch of road is 75 miles per hour, then the amount of time it takes a car going x miles per hour ~~over the limit to~~ travel the stretch is given by $f(x) = \frac{100}{75+x}$

a. How long does it take the car to travel the stretch if the car is going 10 miles per hour over the limit?

$f(10) = \frac{100}{75+10} = \frac{100}{85} \approx 1.17$ hrs.

b. How long does it take the car to travel the stretch if the car is not speeding at all?

Initial $f(0) = \frac{100}{75+0} = \frac{100}{75} = 80$ M.P.

Domain of a Function

Recall that the **domain** of a function is the set of all **inputs**. Domain may be written **explicitly**. For example, for the function $f(x) = x^2, \quad 0 \le x \le 5$, the domain is specifically set as all inputs between and including 0 and 5. Hence its domain is simply $[0, 5]$.

~~less~~ than, ~~not~~ equal to
greater " or equal to

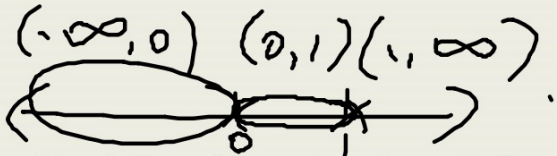
Whenever we have a function without the domain stated explicitly, we need to figure it out by algebraic reasoning.

Ex. $f(x) = x^2 + 1$ $\quad g(x) = \frac{1}{x-4}$ $\quad h(x) = \sqrt{x}$

$(-\infty, \infty)$

$x - 4 = 0$
$x \neq 4$
D: $x \neq 4$
$(-\infty, 4) \cup (4, \infty)$

$x \geq 0$
D: $x \geq 0$
$[0, \infty)$



Examples

Find the domain of each function.

a. $f(x) = \frac{1}{x^2 - x}$ $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

$x^2 - x = 0$

$x(x-1) = 0$

$\therefore x \neq 0, 1$

b. $g(x) = \sqrt{9 - x^2}$

c. $h(t) = \frac{t}{\sqrt{t+1}}$

Four Ways of Representing a Function

To help us understand what a function is, we have used machine and arrow diagrams. We can represent a functional relationship in following ways:

1. Verbally (by a description in words)

2. Algebraically (by an explicit formula)

3. Visually (by a graph)

4. Numerically (by a table of values)

Four Ways to Represent a Function

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Verbal

Using words:

$P(t)$ is "the population of the world at time t"

Relation of population P and time t

Algebraic

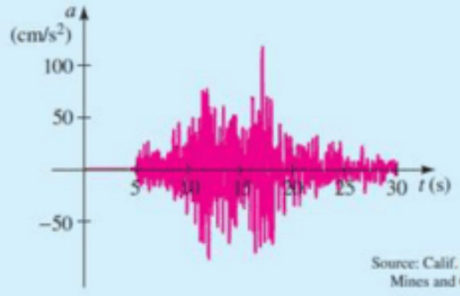
Using a formula:

$$A(r) = \pi r^2$$

Area of a circle

Visual

Using a graph:



Source: Calif. Dept. of Mines and Geology

Vertical acceleration during an earthquake

Numerical

Using a table of values:

w (ounces)	$C(w)$ (dollars)
$0 < w \leq 1$	0.37
$1 < w \leq 2$	0.60
$2 < w \leq 3$	0.83
$3 < w \leq 4$	1.06
$4 < w \leq 5$	1.29
⋮	⋮

Cost of mailing a first-class letter