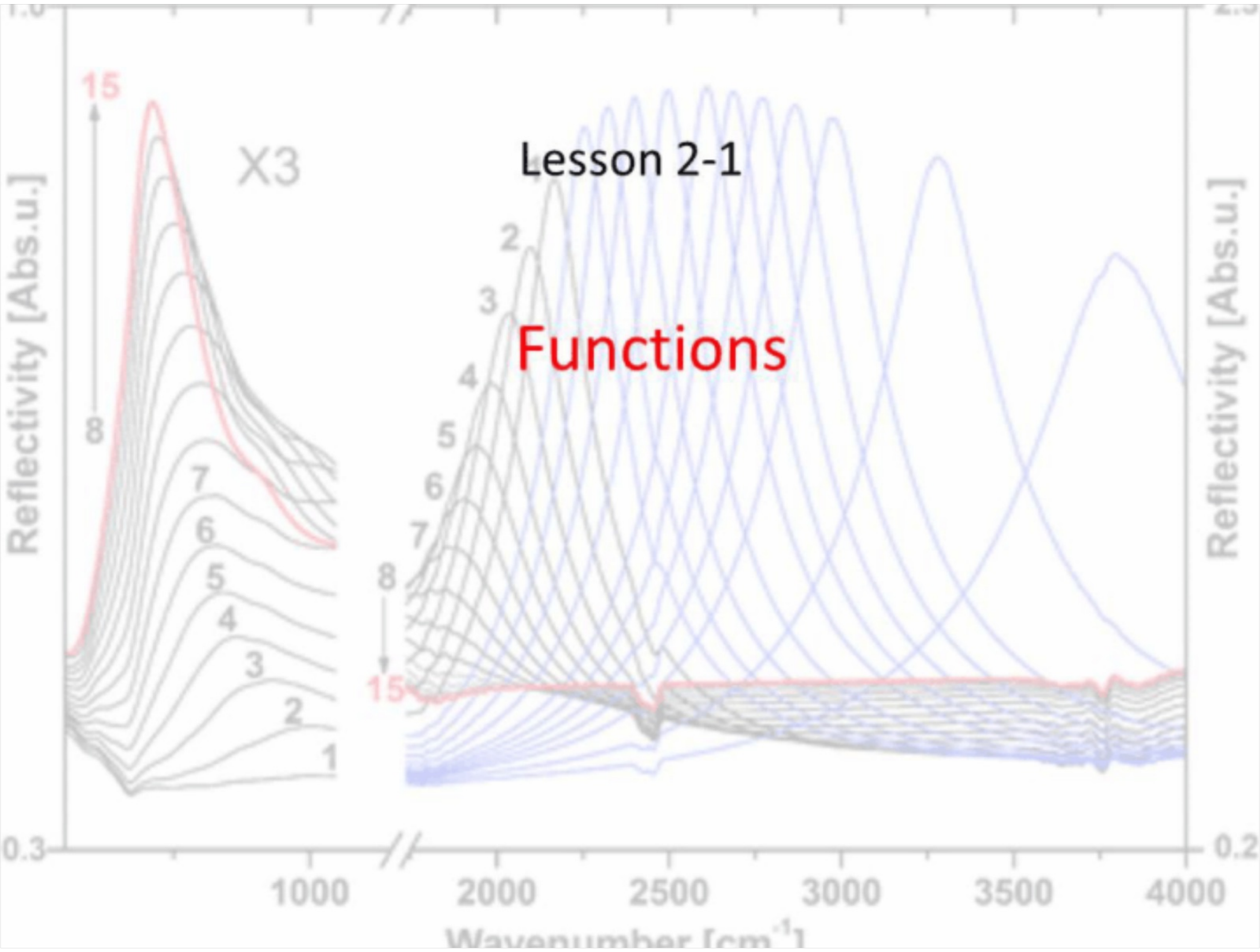
Lesson 2-1

Functions



Objective

Students will…

- Be able to define what an input and an output is.
- Be able to define what a function is.

Functional Relationship

A **functional relationship** is a relationship in which one quantity **depends** on another. In other words, given two variables, one is always **dependent** on the other.

ex. Height is a function of age

Temperature is a function of date

Cost of mail is a function of weight.

Independent vs Dependent Variables

That being said, we must always be able to define both the independent ("plug in") and dependent ("result") variables.

indep. (x, y) depend

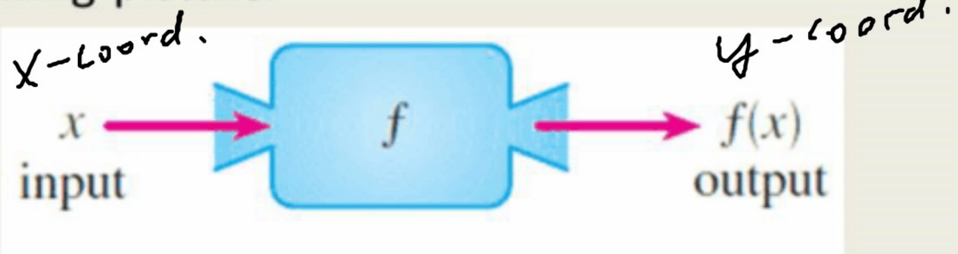
ex. Height is a function of age.

Temperature is a function of date.

Cost of mail is a function of weight.

Input vs Output

Mathematically speaking, we can also differentiate the independent and the dependent variables as inputs and outputs. Consider the following picture:



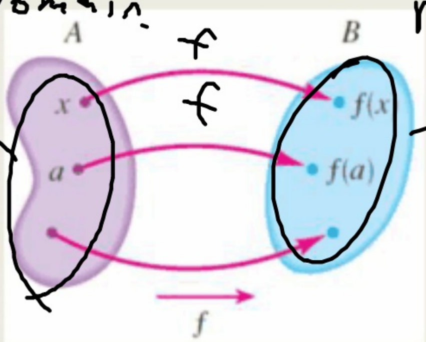
Here the function "f" is the rule that the machine operates in, and what comes out depends on what goes in.

Definition of a Function

So now we are ready to define what a function is.

A **function**, say f, is a rule that assigns to each element (item) x in a certain set A **exactly one** element, called $f(x)$, in a set B.

Ex.



The set A is also known as the **domain**, and set B is known as the **range**.

Examples of Functions

Another way to define function is for every **input**, there is exactly **one** **output**.

Ex.

$f(x) = x - 3$

$f(1) = 1 - 3 = -2$

$f(0) = 0 - 3 = -3$

$f(x) = x^2$

$f(1) = 1^2 = 1$

$f(-1) = (-1)^2 = 1$

Evaluating Functions

Consider the function $f(x) = x - 3$

Here, x is the input, while $f(x)$ is the output. That being said, $f(x)$ would change as x changes. We can evaluate functions by placing different inputs. For the above function,

$$f(a) = \boxed{a - 3}$$

$$f(1) = (1) - 3 = -2$$
$$f(2) = (2) - 3 = -1$$
$$f(0) = (0) - 3 = -3$$
$$f(-3) = (-3) - 3 = -6$$

Examples

Let $f(x) = 3x^2 + x - 5$. Evaluate each function value.

1. $f(-2)$

5

2. $f(0)$

-5

3. $f(4)$

47

4. $f(\frac{1}{2})$

$3(1/4)$

$3/4 + 1/2$

$\frac{5}{4}$

$-\frac{15}{4} =$

Piecewise Functions

Piecewise functions are combination of functions that are defined by the range of inputs.

$C(20) =$

$C(399)$

Ex.
$$C(x) = \begin{cases} 39 & \text{if } 0 \le x \le 400 \\ 39 + 0.2(x - 400) & \text{if } x > 400 \end{cases}$$

$C($

So whenever x is in between or equal to 0 and 400, then the output is always 39. Whenever x is strictly above 400, the bottom function applies.

Examples

Evaluate.

22. $f(x) = \begin{cases} 5 & \text{if } x \le 2 \\ 2x - 3 & \text{if } x > 2 \end{cases}$

$f(-3), f(0), f(2), f(3), f(5)$

$f(-3) = 5$ $f(3) = 3$
$f(0) = 5$ $f(5) = 7$
$f(2) = 5$

Examples

Use the function to evaluate the indicated expression.

$f(x) = 3x - 1;\ f(2x),\ 2f(x)$

$f(2x) = 3(2x) - 1$

$= 6x - 1$

$2(f(x)) = 2(3x - 1)$

$= 6x - 2$

Examples

Find $f(a), f(a+h)$, and the difference quotient $\frac{f(a+h)-f(a)}{h}$

$f(x) = x^2 + 1$

Homework 9/3

TB pg. 155 #1-4, 12, 16, 24, 28, 34