Lesson 1[illegible]-2 ?

Limits II

Objective

Students will…

- Be able to know and use the Limit Laws.
- Be able to find limits by direct substitution.

ex. $\lim_{x\to 2}(3x^2+2x-1)(x^4+9)$ $= \lim_{x\to 2}(3x^2+2x-1)\cdot\lim_{x\to 2}(x^4+9)$

Limit Laws

In this section we seek to find limits algebraically. First off, let's go ahead and look at some properties of limits, called the **Limit Laws**.

Limit Laws

Suppose that c is a constant and that the following limits exist:

$$\lim_{x\to a} f(x) \qquad \text{and} \qquad \lim_{x\to a} g(x)$$

Then

1. $\lim_{x\to a}[f(x)+g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$ — Limit of a Sum

2. $\lim_{x\to a}[f(x)-g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$ — Limit of a Difference

3. $\lim_{x\to a}[cf(x)] = c\lim_{x\to a} f(x)$ — Limit of a Constant Multiple

4. $\lim_{x\to a}[f(x)g(x)] = \lim_{x\to a} f(x)\cdot\lim_{x\to a} g(x)$ — Limit of a Product

5. $\lim_{x\to a}\dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ if $\lim_{x\to a} g(x) \neq 0$ — Limit of a Quotient

$\lim_{x\to 3} 2x^3$

$2\left(\lim_{x\to 3} x^3\right)$

$\lim_{x\to 3} 2x^3-4x^2+7$

$= \lim_{x\to 3} 2x^3 - \lim_{x\to 3} 4x^2 + \lim_{x\to 3} 7$

$y = 9$

Limit Laws Cont.

Limit Laws

$\lim_{x \to 2} (x^2 - 9x)^7 = \left(\lim_{x \to 2} x^2 - 9x\right)^7$

6. $\lim_{x \to a} [f(x)]^n = [\lim_{x \to a} f(x)]^n$ where n is a positive integer — Limit of a Power

7. $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)}$ where n is a positive integer — Limit of a Root

[If n is even, we assume that $\lim_{x \to a} f(x) > 0$.]

Some Special Units

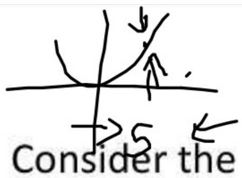
c = constant

1. $\lim_{x \to a} c = c$ ex. $\lim_{x \to 1000264 9 \times 10^{-149}} 9 = 9$

2. $\lim_{x \to a} x = a$

3. $\lim_{x \to a} x^n = a^n$ where n is a positive integer

4. $\lim_{x \to a} \sqrt[n]{x} = \sqrt[n]{a}$ where n is a positive integer and $a > 0$



Direct Substitution

$\lim_{x \to 4} \frac{1}{\sqrt{x} - 1} = \frac{1}{\sqrt{4} - 1} = 1$

Consider the previous problem that we ~~just~~ did

$$\lim_{x \to 5} (2x^2 - 3x + 4)$$

What's interesting here is that $f(5) = 39$, which was indeed the limit from above. It turns out this can be generalized.

Limits by Direct Substitution: If f is a polynomial or a rational function and a is in the domains of f, then

$$\lim_{x \to a} f(x) = f(a) \qquad \text{and} \qquad \lim_{x \to a} \frac{g(x)}{h(x)} = \frac{g(a)}{h(a)}$$ provided, $h(a) \neq 0$

Functions with this property are called **continuous at a**. You will learn more about this in 1.4. For any continuous function, similar idea applies.

Example

a. $\lim_{x \to -2} \frac{x^3+2x^2-1}{5-3x} = \frac{(-2)^3+2(-2)^2-1}{5-3(-2)} = \frac{\cancel{-8+8}-1}{5+6} = \boxed{-\frac{1}{11}}$

$\lim_{x \to 5/3}$

b. $\lim_{x \to 0} \frac{x-2}{x^2+x-6} = \frac{0-2}{0^2+0-6}$

$= 1/3$

Example

c. $\lim_{x \to 3}(2x^3 - 10x - 8) = 16$

d. $\lim_{x \to -1} \frac{x^2+5x}{x^4+2} = \frac{-4}{3}$

Algebra and Direct Substitution

Indeterminate case.

Evaluating limits by direct substitution is easy. But not all limits can be evaluated this way. In fact, most of the situations in which limits are useful requires us to work harder to evaluate the limit. Consider…

$$\lim_{x \to 1} \frac{x-1}{x^2-1} = \lim_{x \to 1} \frac{1-1}{1-1} = \frac{0}{0}?$$

Here, we can't just use direct substitution because we end up with a zero on the denominator. We would need to use algebra. There are generally three major ways.

Cancelling out a Common Factor

Our previous example: $\lim_{x\to 1} \frac{x-1}{x^2-1}$, needs to be done by factoring.

$$\lim_{x\to 1} \frac{x-1}{x^2-1} = \lim_{x\to 1} \frac{\cancel{x-1}}{(x+1)\cancel{(x-1)}} = \lim_{x\to 1} \frac{1}{x+1} = \frac{1}{1+1} = \frac{1}{2}$$

Finding Limits by Simplifying

Some limits can be found by simply…simplifying!

Ex. $\lim_{h\to 0}\frac{(3+h)^2-9}{h} = \lim_{h\to 0}\frac{\cancel{9}+6h+h^2\cancel{-9}}{h} = \lim_{h\to 0}\frac{6h+h^2}{h} = \lim_{h\to 0}\frac{\cancel{h}(6+h)}{\cancel{h}}$

$= 6+0$

$= 6$

Finding Limits by Rationalizing

Some limits require rationalizing. ← "Mathematicians' greatest trick".

Ex. $\lim_{t\to 0}\left(\frac{\sqrt{t^2+9}-3}{t^2}\right)\cdot\left(\frac{\sqrt{t^2+9}+3}{\sqrt{t^2+9}+3}\right) = \lim_{t\to 0}\left(\frac{t^2+\cancel{9}-\cancel{9}}{t^2\left(\sqrt{t^2+9}+3\right)}\right) = \frac{\cancel{t^2}}{\cancel{t^2}\left(\sqrt{t^2+9}+3\right)}$

$= \lim_{t\to 0}\frac{1}{\sqrt{t^2+9}+3} = \frac{1}{\sqrt{9}+3} = \boxed{\frac{1}{6}}$

Homework 9/5

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