Lesson 2-2

Derivative II

Objective

Students will…

- Be able to know and use the basic differentiation rule.
- Be able to use the derivatives to find rates of change.
- Be able to relate derivative function to the velocity function.

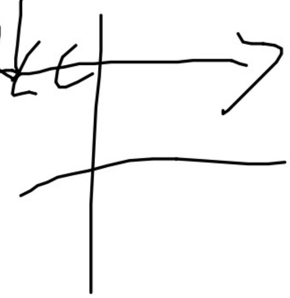
Constant Rule

You now know what differentiation is, and find derivative functions. As always in mathematics, it is always a good thing if certain things can be generalized into an easier or simpler form. The first and foremost, the most basic result of differentiation is none other than the **constant rule**.

The derivative of a constant function is 0. That is, if c is a real number, then

$$\frac{d}{dx}[c] = 0$$

$$\frac{d}{dx}c = \lim_{h \to 0}\left(\frac{c - c}{h} = \frac{0}{h} = 0\right) \Rightarrow \lim_{h \to 0} 0 = \boxed{0}$$



Think the graph of any constant function. The slope is **ALWAYS** zero.

The Power Rule

The most important and useful rule in derivative would be the **power rule**.

The Power Rule- If n is a rational number, then the function $f(x) = x^n$ is differentiable and $\frac{d}{dx}[x^n] = nx^{n-1}$

☆ For f to be differentiable at $x = 0$, n must be a number such that x^{n-1} is defined on an interval containing 0.

$(x+h)^2 = 1x^2h^0 + \boxed{2}x^1h^1 + 1x^0h^2$

$(x+h)^3 = 1x^3h^0 + \boxed{3}x^2h^1 + 3x^1h^2 + 1x^0h^3$

$(x+h)^4 = 1x^4h^0 + \boxed{4x^3h^1} + \cdots$

$nx^{n-1}h$

Proof of the Power Rule

$$\frac{d}{dx}x^n = \lim_{h\to 0}\left(\frac{(x+h)^n - x^n}{h} = \frac{\cancel{x^n} + nx^{n-1}h^1 + \cdots + h^n - \cancel{x^n}}{h}\right)$$

$$= \ldots\left(\frac{\cancel{h}\left(\boxed{nx^{n-1}} + \cdots + h^{n-1}\right)}{\cancel{h}}\right) = \boxed{nx^{n-1}}$$

Example

Find the derivative of the following:

a. $f(x) = x^3$

$f'(x) = \boxed{3x^2}$

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \frac{(x+h)^3 - x^3}{h}$

$\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x^3}$

$= \frac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}} = \boxed{3x^2} + 3xh + h^2$

b. $g(x) = \sqrt[3]{x} = x^{1/3}$

$= \boxed{\frac{1}{3}x^{-2/3}}$

$= \boxed{\frac{1}{3\sqrt[3]{x^2}}}$

x^n

Example

Find the derivative of the following:

c. $y = \frac{1}{x^2} = 1x^{-2} \Rightarrow y' = -2x^{-3} = -\frac{2}{x^3}$

$\cancel{\frac{1}{2x}}$

Example

Find the equation of a tangent line to the graph of $f(x) = x^2$ when $x = -2$. $(-2, 4)$

$f'(x) = 2x$

$f'(-2) = 2(-2) = -4 = m$

$\boxed{y - 4 = -4(x+2)}$

Laws Derivatives

1. $\frac{d}{dx}[cf(x)] = cf'(x)$, where c is a real number.

ex. $y = 2x^2 \Rightarrow y' = 4x = 2(2x) = 2(y' = x^2)$

2. $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)] = f'(x) \pm g'(x)$

ex. $y = 3x^7 + 6x^6 + 5x^3 - 21x^2 + 1001$

$\boxed{y' = 21x^6 + 36x^5 + 15x^2 - 42x}$

*<u>**NOTE**</u>: $\frac{d}{dx}[f(x) \times \div\, g(x)] \neq f'(x) \times \div\, g'(x)$

Trig Derivatives

1. $\frac{d}{dx}[\sin x] = \cos x$

2. $\frac{d}{dx}[\cos x] = -\sin x$

Avg vs Instantaneous Rate of Change

Derivative.

Ex. If a billiard ball is dropped from a height of 100 feet, its height s at time t is given by the position function: $s = -16t^2 + 100$, where s is measured in feet and t is measured in seconds. Find the average velocity (rate of change) over each of the following time intervals.

a. $[1, 2]$

$x_1 \quad x_2$

$(1, 84)$

$(2, 36)$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{36 - 84}{2 - 1} = \boxed{-48}$$

b. $[1, 1.5]$

Avg vs Instantaneous Rate of Change

Ex. At time $t = 0$, a diver jumps from a platform diving board that is 32 feet above the water. The position of the diver is given by

$$s(t) = -16t^2 + 16t^1 + 32,$$

$t = -1, \boxed{2}$

where s is measured in feet and t is measured in seconds.

a. When does the diver hit the water?

$0 = -16t^2 + 16t + 32 = -16(t^2 - t - 2) = (t+1)(t-2)$

b. What is the diver's velocity at impact?

$s'(t) = -32t + 16$

$s'(2) = -32(2) + 16$
$= -64 + 16 = \boxed{-48}$

Homework 9/27

2.2 Exercises #3-23 (odd), 39-51 (odd), 93, 94, 103, 104