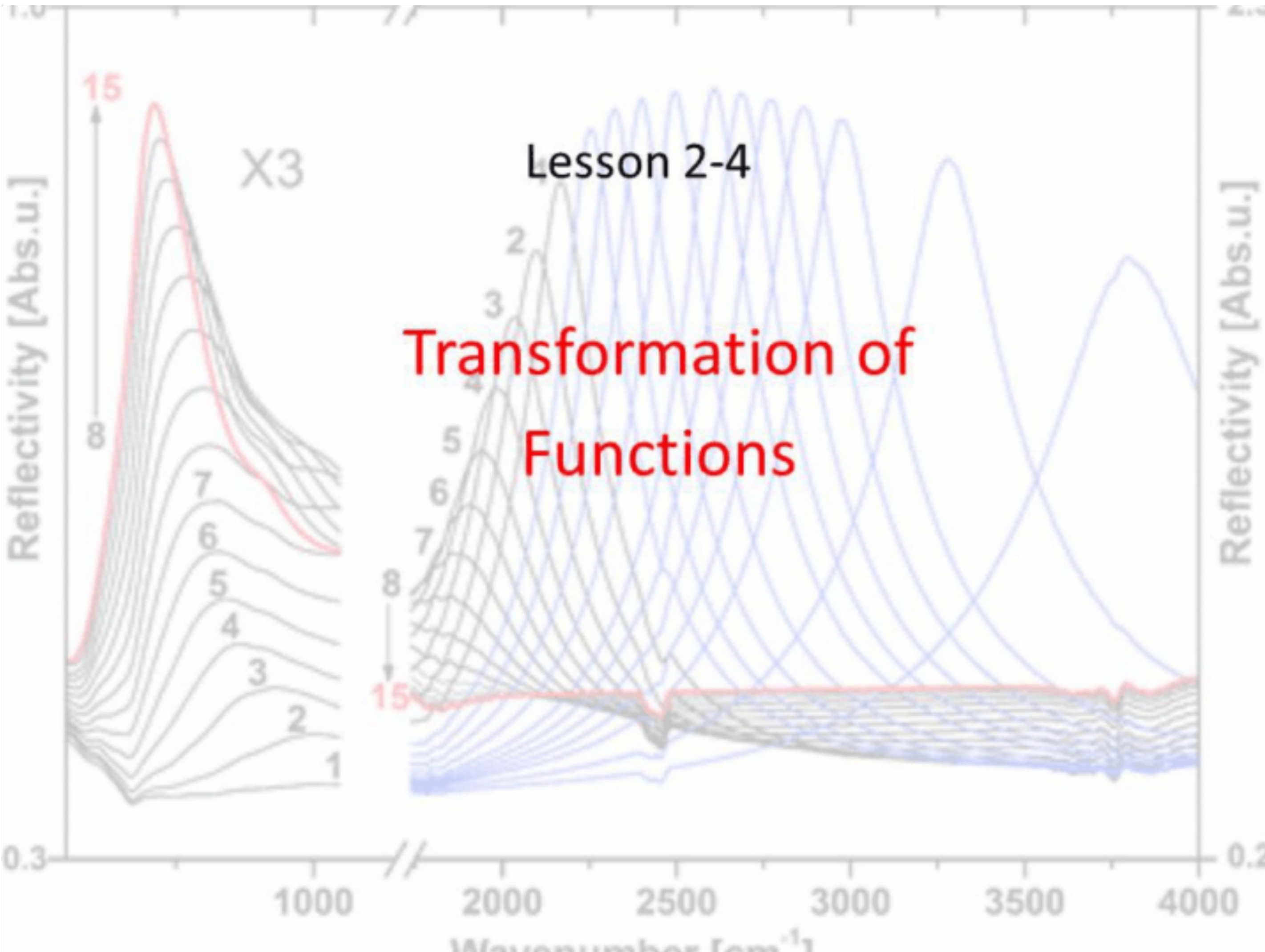
Warm Up 9/11

Describe the shift of the function: $g(x) = (x + 11)^2 - 2$ from its "parent" function, $f(x) = x^2$

Describe the shift of the function $h(x) = (x - 6)^5 + 1$ from its "parent" function, $f(x) = x^5$

Lesson 2-4

Transformation of Functions



Objective

Students will…

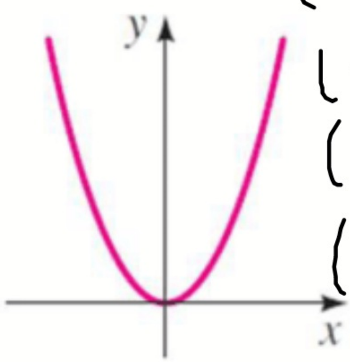
- Be able to apply the properties of <u>reflections</u> in graphing various functions.
- Be able to determine whether a function is even or odd.

“Parent” Functions

We have seen and studied some of the standard functions and their graphs. For example.

$f(x) = x^2$

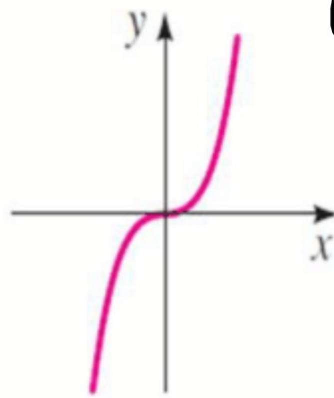
(-2, 4)
(-1, 1)
(0, 0)
(1, 1)
(2, 4)



$f(x) = x^2$

$f(x) = x^3$

(-2, -8)
(-1, -1)
(0, 0)
(1, 1)
(2, 8)



$f(x) = x^3$

$-2^2 = 4$

$(-2)^2 = 4$

Transformation of Functions

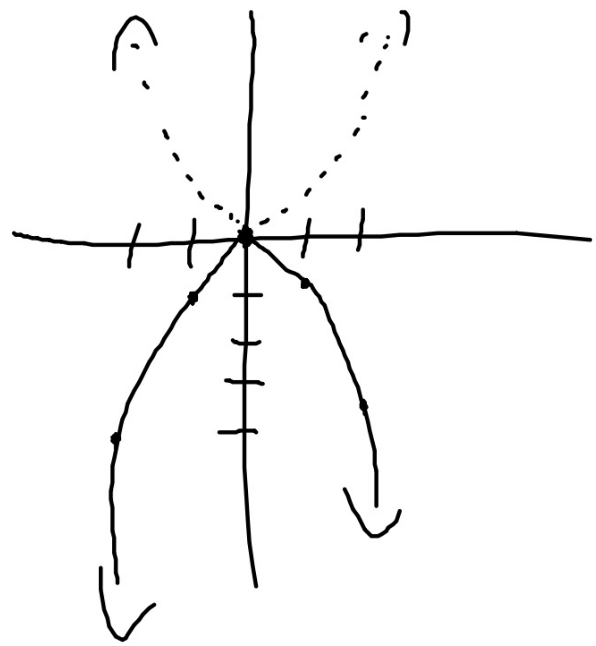
$3^2 = 9$

$-3^2 = -9 \quad -(3^2)$

$(-3)^2 = 9$

Let's go ahead and compare the two functions: $f(x) = x^2$ and $g(x) = -(x^2)$

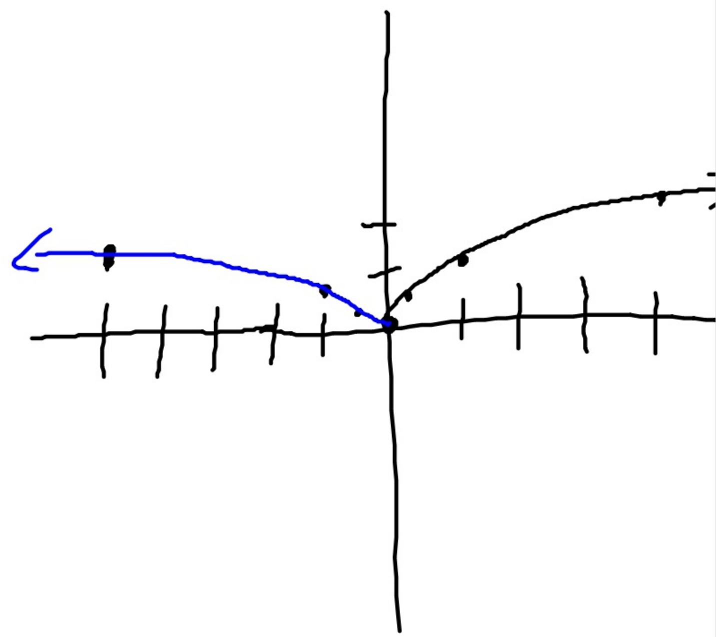
x^2	$-x^2$
$(-2, 4)$	$(-2, -4)$
$(-1, 1)$	$(-1, -1)$
$(0, 0)$	$(0, 0)$
$(1, 1)$	$(1, -1)$
$(2, 4)$	$(2, -4)$



Transformation of Functions

Now let's compare the functions: $f(x) = \sqrt{x}$ and $g(x) = \sqrt{-x}$

$\sqrt{x}$	$\sqrt{-x}$
$(0,0)$	$(0,0)$
$(1,1)$	$(-1,1)$
$(4,2)$	$(-4,2)$
$(1/4, 1/2)$	$(-1/4, 1/2)$



Transformation: Reflection

As observed, the differences between the two functions were either **horizontal or vertical** reflection. This can be generalized by the following:

Along the y-axis (horizontal)

$y = f(-x)$ reflects the graph of $y = f(x)$ along the y-axis (horizontal reflection).

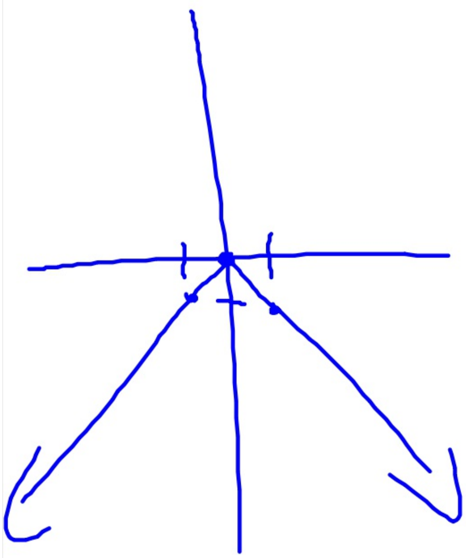
Along the x-axis (vertical)

$y = -f(x)$ reflects the graph of $y = f(x)$ along the x-axis (vertical reflection).

Examples

Sketch the following functions by transforming its "parent" function.

a. $f(x) = -|x|$



b. $f(x) = (-x)^3$

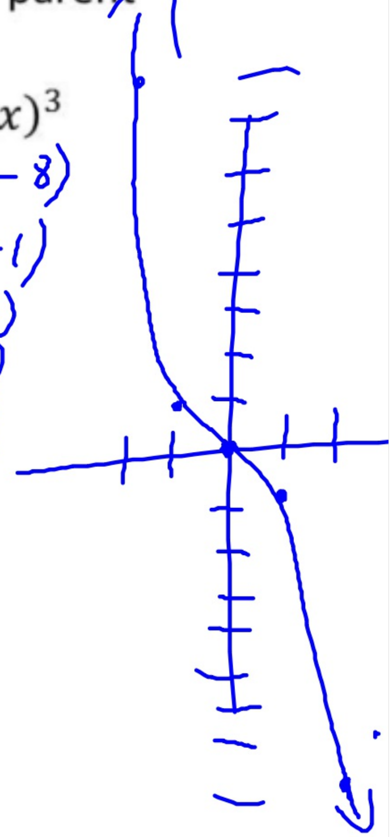
$(-2,-8) \rightarrow (2,-8)$

$(-1,-1) \rightarrow (1,-1)$

$(0,0) \rightarrow (0,0)$

$(1,1) \rightarrow (-1,1)$

$(2,8) \rightarrow (-2,8)$



Even Functions

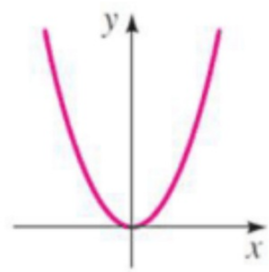
Consider the function $f(x) = x^2$. We observed that it can be reflected vertically, i.e. along the x-axis. What happens when we try to reflect this function horizontally, i.e. along the y-axis?

This would mean that the equation would be written in the form of

$$f(x) = (-x)^2 = x^2$$

So, as you can see horizontal reflection really did not change anything. This can easily be seen on its graph.

Any function that has this characteristic is called an **<u>even</u>** function.

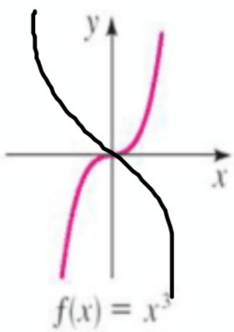


$f(x) = x^2$

Odd Functions

$(-x)^3 = -(x^3)$

Now consider the function $f(x) = x^3$. We have already seen it reflected horizontally, i.e. along the y-axis. What happens when we reflect this graph vertically, i.e. along the x-axis? Look at the graph!



Here the graph looks the same whether it is reflected vertically or horizontally. This can easily be seen algebraically: $(-x)^3 = -(x^3)$.

Any function that has this characteristic is called an **odd** function.

Even and Odd Functions

So now we give a formal, generalized definition of even and odd functions:

Let f be a function,

f is even if $f(-x) = f(x)$, for all x in the domain of f

f is odd if $f(-x) = -f(x)$, for all x in the domain of f

Ex. Determine whether the following functions are even or odd.

a. $f(x) = x^5 + x$ b. $g(x) = 1 - x^4$ c. $h(x) = 2x - x^2$

$f(-x) = (-x)^5 + (-x)$
$= -x^5 - x$
$= -(x^5 + x)$
$= -f(x)$

Odd

$g(-x) = 1 - (-x)^4$
$g(-x) = 1 - x^4$
$g(-x) = g(x)$

even

$h(-x) = 2(-x) - (-x)^2$
$= -2x - x^2$
$= -(2x + x^2)$

Neither

Homework 9/11

TB pg. 190 #16, 35, 36, 40, 61-68