Warm Up 9/10

1. Define function

   For every input there is exactly one output.

2. Evaluate $f(0)$ and $f(2)$ for the following.

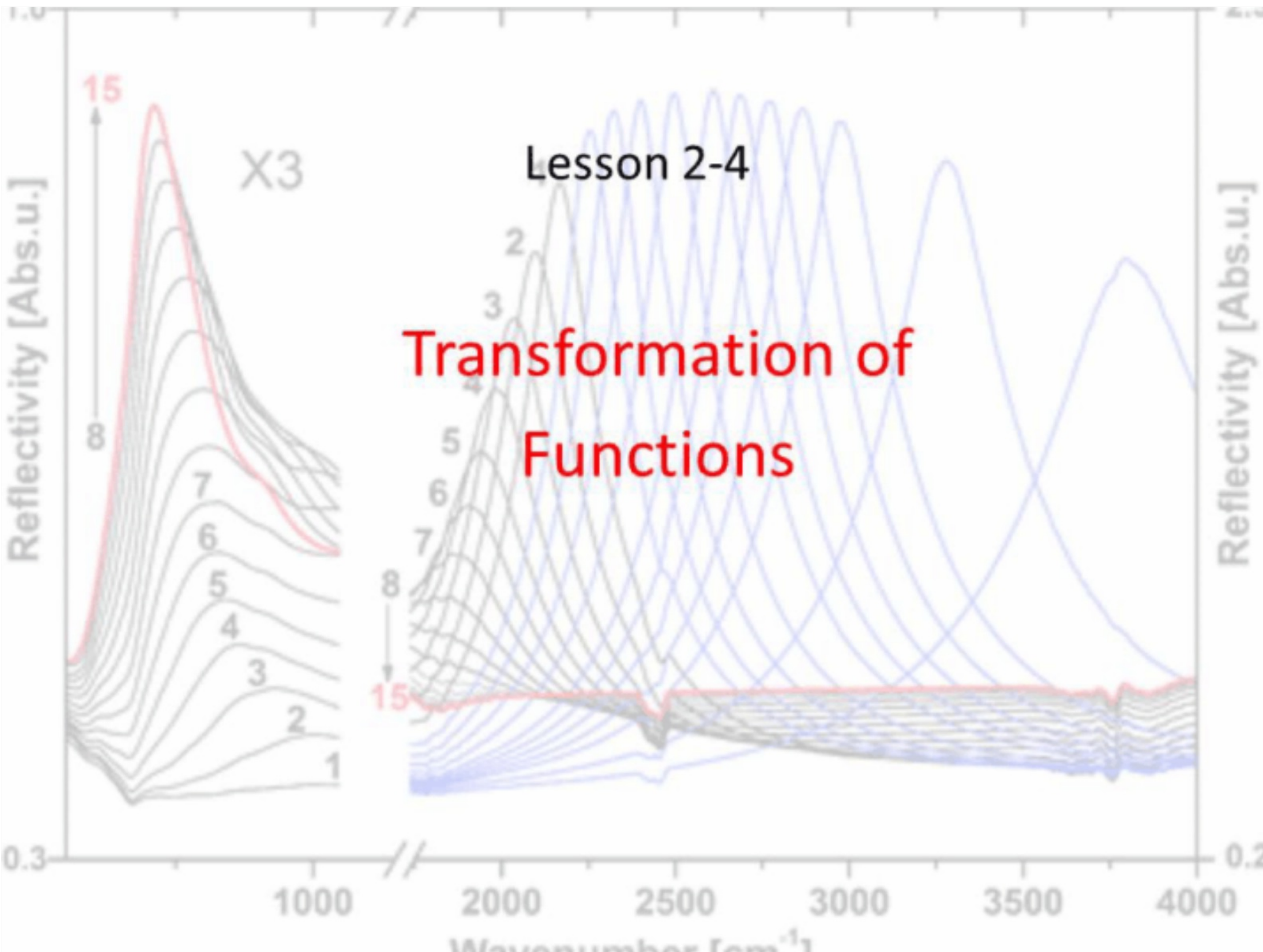
   a. $f(x) = x^2$ b. $g(x) = x^2 - 2$

   $f(0) = 0 \longrightarrow g(0) = -2$

   $f(2) = 4 \longrightarrow g(2) = 2$

Lesson 2-4

Transformation of Functions



Objective

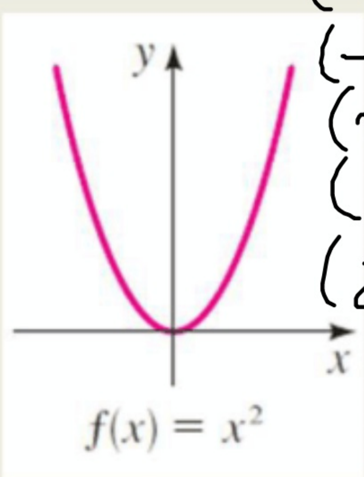
Students will…

- Be able to understand the basic idea of transformation of functions.
- Explore and apply the properties of vertical and horizontal shifts.

“Parent” Functions
mother

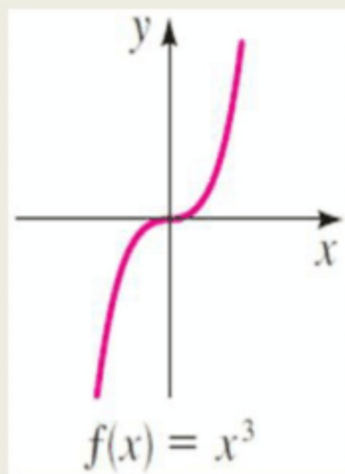
We have seen and studied some of the standard functions and their graphs. For example.

$f(x) = x^2$



(−2, 4)
(−1, 1)
(0, 0)
(1, 1)
(2, 4)

$f(x) = x^3$



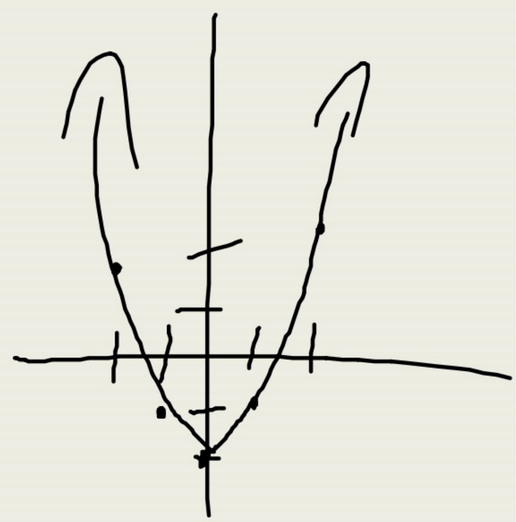
(−2, −8)
(−1, −1)
(0, 0)
(1, 1)
(2, 8)

Transformation of Functions

$g(x) = f(x) - 2$

Now, consider our problem from the warm up. Let's go ahead and compare the two functions: $f(x) = x^2$ and $g(x) = x^2 - 2$

x^2		$x^2 - 2$
$(-2, 4)$		$(-2, 2)$
$(-1, 1)$	y values	$(-1, -1)$
$(0, 0)$	$\xrightarrow{-2}$	$(0, -2)$
$(1, 1)$		$(1, -1)$
$(2, 4)$		$(2, 2)$



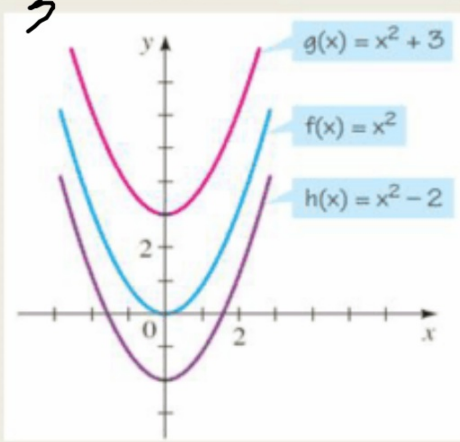
Transformation: Vertical Shift

As observed, the difference between $f(x)$ and $g(x)$ was that $g(x)$ was simply $f(x)$ vertically **shifted** down ~~**up**~~ **2 units**. This can be generalized by the following:

$y = f(x) \pm c$ shifts the graph of $y = f(x)$ upward(+) or downward(-) c units, for $c > 0$.

Ex. Use the graph of $f(x) = x^2$ to sketch the graph of,

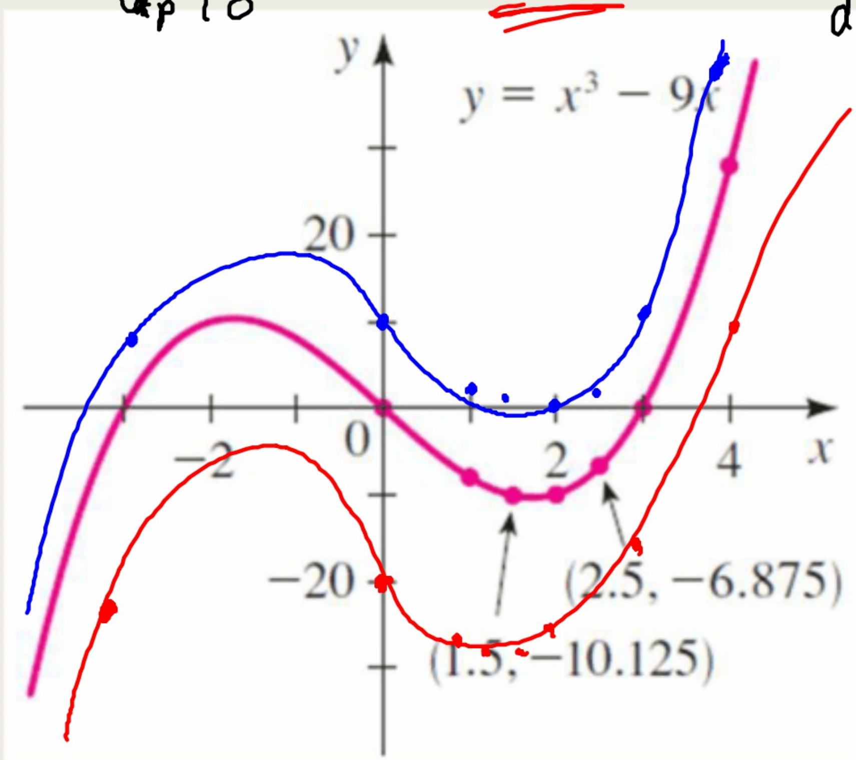
$g(x) = x^2 + 3$ up 3 and $h(x) = x^2 - 2$ down 2



Example

Use the graph of $\boxed{f(x) = x^3 - 9x}$ shown below to sketch the graph of $g(x) = x^3 - 9x + 10$ and $h(x) = x^3 - 9x - 20$

up 10 down 20



Transformation: Horizontal Shift

Similar to vertical shift, we also have a **horizontal shift**. Let's compare the three functions: $f(x) = x^2$, $g(x) = (x+2)^2$, $h(x) = (x-1)^2$

left 2.

x^2	x value	$(x+2)^2$	$(x-1)^2$ right 1.
$(-2,4)$	$\xrightarrow{-2}$	$(-4,4)$	$(-1,4)$
$(-1,1)$	$\xrightarrow{-2}$	$(-3,1)$	$(0,1)$
$(0,0)$		$(-2,0)$	$(1,0)$
$(1,1)$		$(-1,1)$	$(2,1)$
$(2,4)$		$(0,4)$	$(3,4)$

Transformation: Horizontal Shift

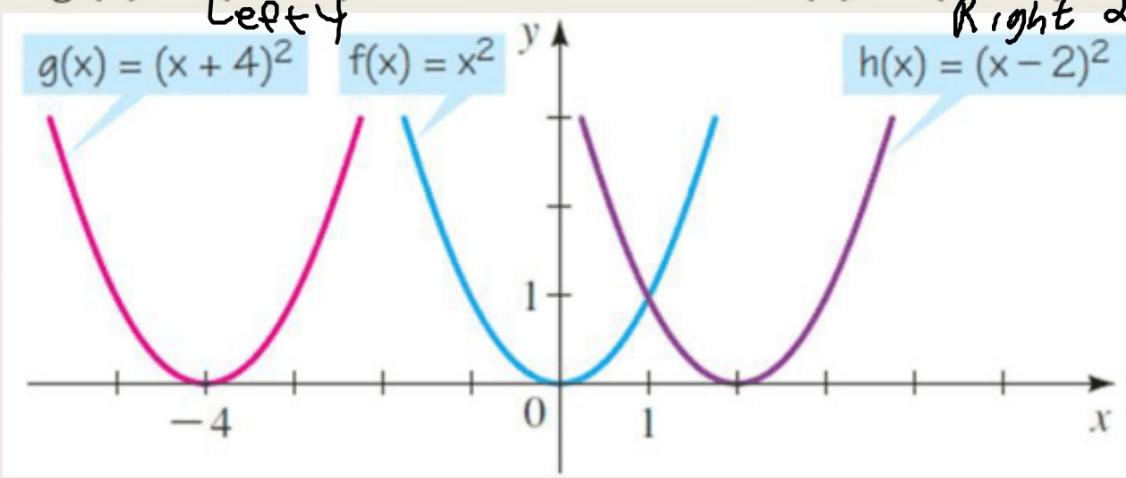
So the horizontal shift can also be generalized.

$y = f(x \pm c)$ shifts the graph of $y = f(x)$ to the right($-$) or left($+$) c units, for $c > 0$. Note the **opposite** signs!

Ex. Use the graph of $f(x) = x^2$ to sketch the graph of,

$g(x) = (x + 4)^2$ and $h(x) = (x - 2)^2$

Left 4 Right 2

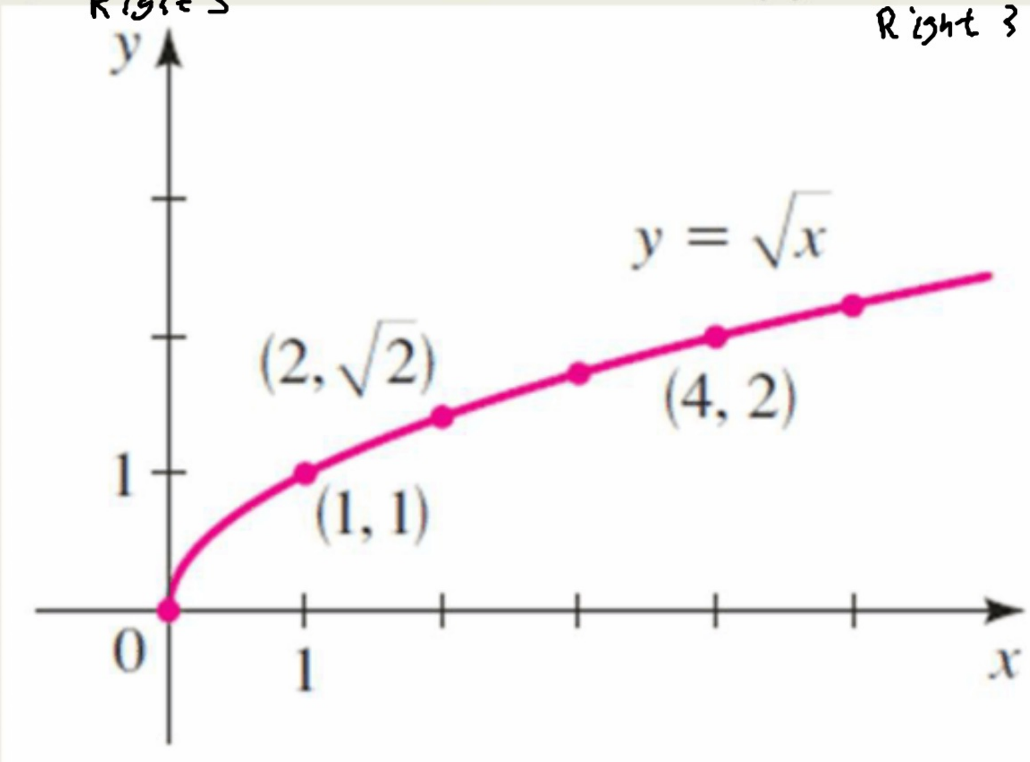


Example

Use the graph of $f(x) = \sqrt{x}$ shown below to sketch the graph of

$g(x) = \sqrt{x-3}$ and $h(x) = \sqrt{x-3} + 4$

Right 3 Right 3 up 4.



Examples

Describe the shift of the function: $g(x) = (x+11)^2 - 2$ from its "parent" function, $f(x) = x^2$

left 11 down 2

$\leftarrow 11$ $\downarrow 2$

Describe the shift of the function $h(x) = (x-6)^5 + 1$ from its "parent" function, $f(x) = x^5$

$\rightarrow 6,\ \uparrow 1$

Describe the shift of the function $p(x) = \sqrt{x+5} - 4$ from its "parent" function, $f(x) = \sqrt{x}$

$\leftarrow 5,\ \downarrow 4$

Homework 9/10

TB pg. 190 #1-3, 7, 11, 13, 19 (a, b, d), 27, 28, 33, 37, 39