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Lesson 2-4

Derivative IV:
The Chain Rule

Objective

Students will…

- Be able to know and use the chain rule.

Derivative of Composite Functions

Recall that a composition of functions, or the composite function $f \circ g$ (also called a composition of f and g) is defined by

$$(f \circ g)(x) = f(g(x))$$

For finding the derivative of any composite function, we need to use the chain rule.

Chain Rule

<u>Chain Rule</u>- If f and g are differentiable functions, then the composite function $f \circ g = f(g(x))$ is also differentiable such that …

outside inside.

$$\frac{d}{dx} f(g(x)) = \boxed{f'(g(x)) \cdot g'(x)}$$

In other words, the chain rule consists of first taking the derivative of the outer function, while leaving the inner function the same, multiplied by the derivative of the inner function.

The most challenging part of the chain rule being able to distinguish different functions that are being composed (i.e. outer and inner).

Example

EXAMPLE 2 Decomposition of a Composite Function

$y = 1 \div (x+1)$

$y = f(g(x))$	$u = g(x)$	$y = f(u)$
a. $y = \dfrac{1}{x+1}$ $\div$	$u = x + 1$	$y = \dfrac{1}{u}$
b. $y = \sin 2x$	$u = 2x$	$y = \sin u$
c. $y = \sqrt{3x^2 - x + 1}$	$u = 3x^2 - x + 1$	$y = \sqrt{u}$
d. $y = \tan^2 x$	$u = \tan x$	$y = u^2$

$= (\tan x)^2$

Example

Find the derivative.

a. $y = (x^2+1)^3 = (x^2+1)(x^2+1)(x^2+1)$

$y' = 3(x^2+1)^2 \frac{d}{dx}(x^2+1)$

$= 3(x^2+1)^2 \cdot 2x = \boxed{6x(x^2+1)^2}$

Example

Find the derivative.

$f(x) = \sqrt[3]{(x^2-1)^2} = (x^2-1)^{2/3}$

$f'(x) = \frac{2}{3}(x^2-1)^{-1/3} \cdot \frac{d}{dx}(x^2-1)$

$\boxed{= \frac{4}{3}x(x^2-1)^{-1/3} = \frac{4x}{3\sqrt[3]{x^2-1}}}$

Example

Find the derivative.

$$g(x) = \frac{-7}{(2t-3)^2} = -7(2t-3)^{-2}$$

$$g'(x) = 14(2t-3)^{-3} \cdot 2$$

$$= \boxed{28(2t-3)^{-3} = \frac{28}{(2t-3)^3}}$$

Example

$f'g + g'f$

Find the derivative.

a. $g(x) = \cos 2x$

$g'(x) = -\sin 2x \cdot 2$

$= \boxed{-2\sin 2x}$

b. $y = (\overset{f}{3x})(\overset{g}{\sin 2x})$

$y' = 3\sin 2x + (2\cos 2x)(3x)$

$= \boxed{3\sin 2x + 6x\cos 2x}$

Example

Sin $\qquad f\,(x)$

$f(x)$

Find the derivative.

$y = \sin^3 4t = (\sin 4t)^3$

$= 3(\sin 4t)^2 \cdot \frac{d}{dx}(\sin 4t)$

$= 3(\sin 4t)^2 \cdot \cos 4t \cdot \frac{d}{dx} 4t$

$= \boxed{12(\sin 4t)^2 \cos 4t}$

$\sin(t+t+t+t) \quad \sin 4t$

$\neq$

$\sin 4t \qquad 4 \sin t.$

$\sin t + \sin t + \sin t + \sin t$

Example

$(\tan(3-t))^3 \qquad \frac{f'g - g'f}{g^2}$

Find the derivative.

$$y = \frac{\overset{f}{\tan^3(3-t)}}{\underset{g}{11x^2}} = \frac{\left(3\tan^2(3-t)\cdot \sec^2(3-t)\cdot -1\right)11x^2 - 22x\left(\tan^3(3-t)\right)}{121x^4}$$

$$= \frac{-11x\left(x\left(3\tan^2(3-t)\sec^2(3-t)\right) + 2\left(\tan^3(3-t)\right)\right)}{}$$

$$= \frac{-\tan^2(3-t)\left(3x\sec^2(3-t)\right) + 2\tan(3-t)}{11x^3}$$

Example

Find an equation of the $\boxed{\text{tangent line}}$ to the graph of @ $(\pi, 1)$

$$f(x) = 2\sin x + \cos 2x$$

$$f'(x) = 2\cos x - 2\sin 2x$$

$$= 2(\cos x - \sin 2x)$$

$$f'(\pi) = 2(\underset{-1}{\cos\pi} - \underset{0}{\sin 2\pi}) = -2 = m$$

$$\boxed{y - 1 = -2(x - \pi)}$$

Homework 10/4

2.4 Exercises #1-6, 7-35 (e.o.o), 45-65 (e.o.o), 80, 82, 91, 95