$\frac{6}{4} = \frac{-6}{-4}$

Warm Up 9/9

Find the slope of the line passing through the given points.

a. $(1, 2), (5, 8)$

$m = \frac{-(y_2 - y_1)}{-(x_2 - x_1)} = \frac{6}{4}$

$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{\cancel{-}6}{\cancel{-}4}$

b. $(-1, 3), (0, 2)$

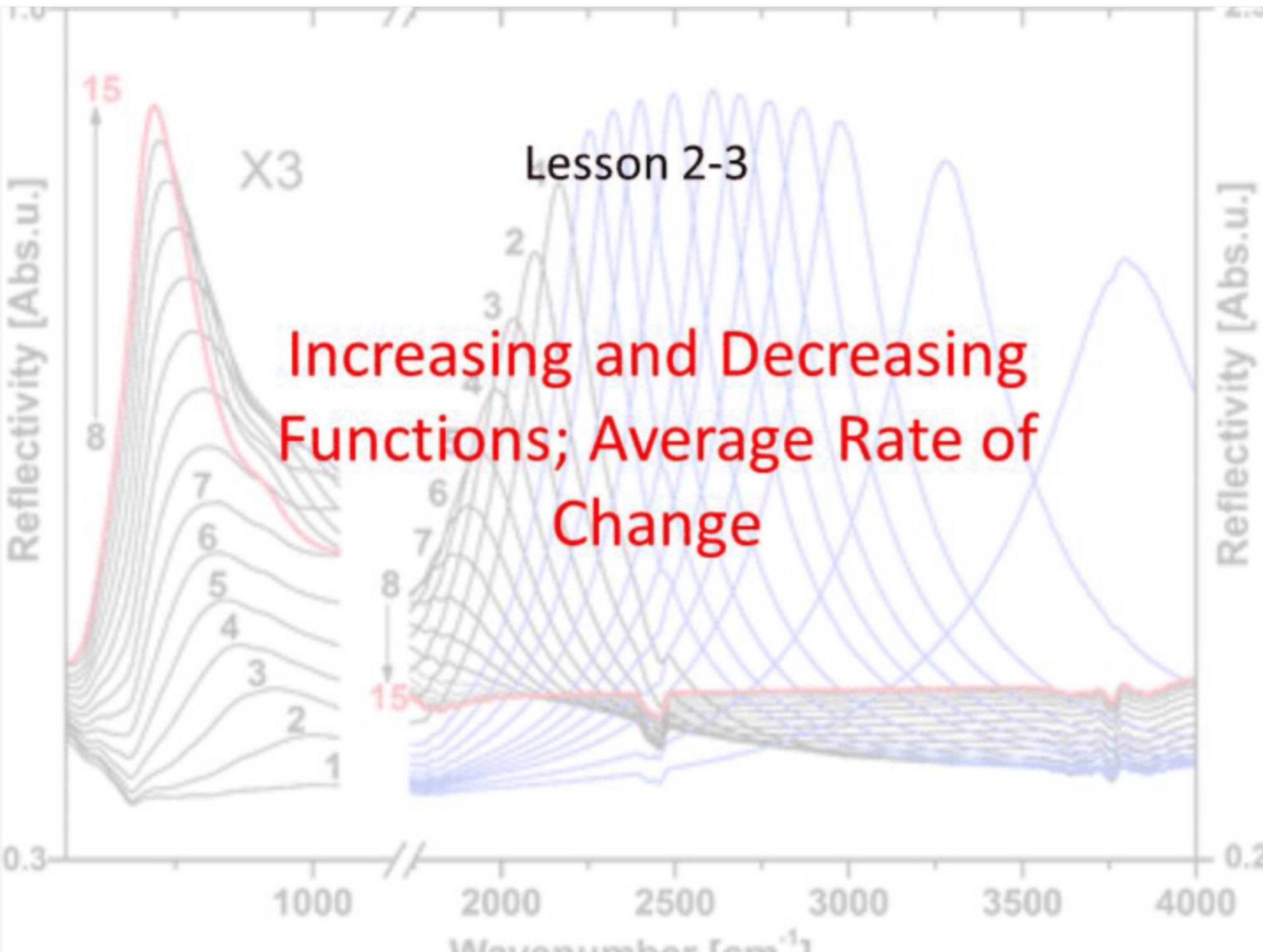
$m = \frac{1}{-1} = -1$

c. $(0, 0), (-11, 7)$

$m = \frac{7}{-11}$

Lesson 2-3

Increasing and Decreasing Functions; Average Rate of Change



Objective

Students will…

- Be able to determine whether a function is increasing or decreasing algebraically and using graphs.
- Be able to compute the average rate of change, and understand its relationship to the secant line.

Increasing and Decreasing Functions

Functions are often used to model changing quantities. Thus, it's important to see and analyze where a function is **increasing** or **decreasing**.

A function, say f is...

Increasing on an interval I if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ in I.
Decreasing on an interval I if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$ in I.

In other words, when a bigger number is **inputted**, the **output** of an increasing function is greater, while the **output** of a decreasing function is smaller.

Examples

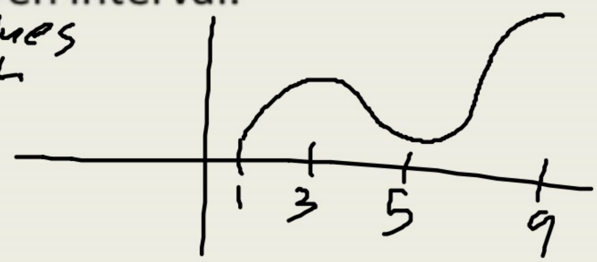
Determine whether the following functions are increasing or decreasing at the given interval. ← x .

a. $f(x) = x + 2$; $[1, 9]$ ← x-values

ex.

$f(1) = 1+2 = 3$

$f(9) = 9+2 = 11$



b. $g(x) = \frac{3}{1+x^2}$; $[-3, 0]$; $[1, 5]$

$g(-3) = \frac{3}{1+9} = \frac{3}{10}$

$g(0) = \frac{3}{1+0} = 3$

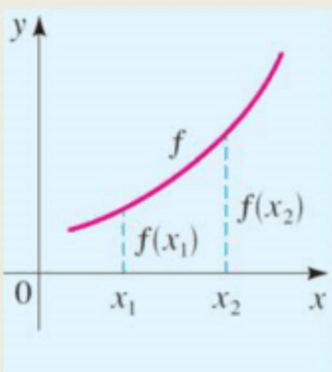
Inc.

$g(1) = \frac{3}{1+1} = \frac{3}{2}$

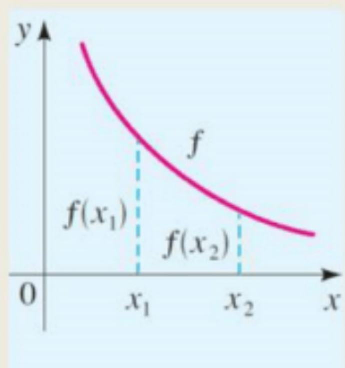
$g(5) = \frac{3}{1+25} = \frac{3}{26}$

Graphs of Increasing and Decreasing Functions

Increasing and decreasing functions can also be easily seen graphically.



f is increasing

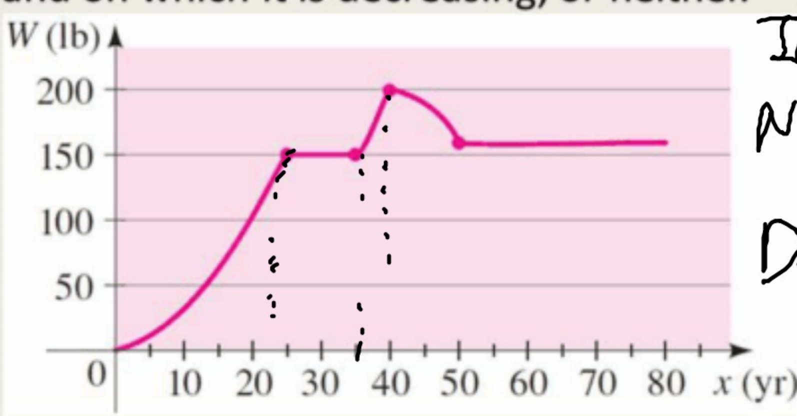


f is decreasing

Thus, when viewing the graph from **<u>left to right</u>**, if the graph is rising the function is increasing, and vice-versa.

Examples

Determine the intervals on which the function W is increasing and on which it is decreasing, or neither.



Inc: [0, 25], [35, 40

Neither: [25, 35], [50, 8

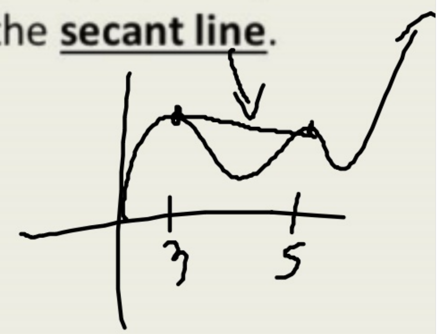
Dec: [40, 50]

Average Rate of Change

Sometimes it is important to find how much a graph has increased or decreased within a certain interval. One of the most useful ways to analyze such change is calculating the **<u>average rate of change</u>**. (ARC) $\frac{f(x_2)-f(x_1)}{x_2-x_1}$

$$\textbf{average rate of change: } \frac{f(b)-f(a)}{b-a} = \frac{\textit{change in } y}{\textit{change in } x} = \frac{y_2-y_1}{x_2-x_1}$$

As you can see the average rate of change is really the **<u>slope</u>** of the line connecting the **<u>two endpoints</u>** of a given interval. This line connecting the two endpoints is known as the **<u>secant line</u>**.



Examples

For the function $f(x) = (x-3)^3$, find the average rate of change between the following intervals:

a. $[\overset{x_1}{1}, \overset{x_2}{3}]$ $\quad ARC = \dfrac{f(3)-f(1)}{3-1}$ $\quad$ b. $[\overset{3}{\cancel{4}}, 7]$ $\quad \dfrac{f(7)-f(3)}{7-4}$ $\quad 16$

$f(1) = (1-3)^3 = -8 \qquad = \dfrac{0 - -8}{3-1} \qquad f(3) = 0 \qquad \dfrac{64}{4} =$

$f(3) = (3-3)^3 = 0 \qquad = \boxed{4}, \qquad f(7) = 64$

For the function $f(x) = (x-3)^3$, find the average rate of change between the interval $[\overset{1}{\cancel{3}}, 7]$.

$f(1) = -8$

$f(7) = 64$

$$\frac{f(7)-f(1)}{7-1} = \frac{64 - -8}{6} = \frac{72}{6} = \boxed{12}$$

Example

If an object is dropped from a tall building, then the distance it has fallen after t seconds is given by the function $d(t) = 16t^2$.
Find its average speed (average rate of change) over the following intervals:

a. $t = 1$ s and $t = 5$ s

$[1, 5]$

$d(1) = 16$

$d(5) = 16(25)$
$= 400$

$\frac{400-16}{4}$

96

b. $t = a$ and $t = a + h$

$[a, a+h]$

$f(a) = 16a^2$

$f(a+h) = 16(a^2 + 2ah + \ldots$

$= 16a^2 + 32ah + \ldots$

$\frac{f(a+h) - f(a)}{\cancel{a} + h - \cancel{a}}$

$\frac{\cancel{16a^2} + 32a\cancel{h} + 16h^{\cancel{2}} - \cancel{16a^2}}{\cancel{h}}$

$= \boxed{32a + 16h}$

Derivatives

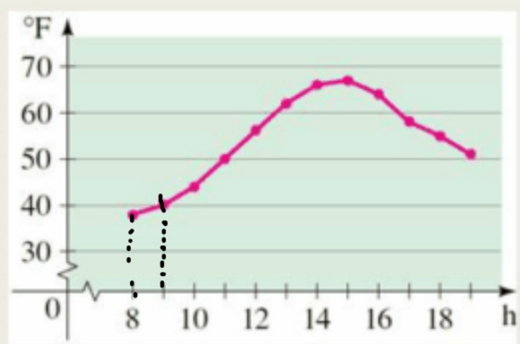
Using the graph of the function of temperature $F(t)$ in given time t, find the average rate of temperature between the following times:

a. 8am to 9am b. 1pm to 3pm c. 4pm to 7pm

$[8, 9]$ $[13, 15]$. $[16, 19]$.

$$\frac{F(9)-F(8)}{9-8}$$

$$\approx \frac{40-38}{1} = \boxed{2}$$



Homework 9/9

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