Warm Up 9/4

Let $f(x) = 2x^2 + 3x - 1$. Evaluate $f(a)$, $f(a+h)$, $f = \frac{f(a+h)-f(a)}{h}$

$f(a) = 2a^2 + 3a - 1$

$f(a+h) = 2(a+h)^2 + 3(a+h) - 1$

$= 2(a^2 + 2ah + h^2) + 3a + 3h - 1$

$= 2a^2 + 4ah + 2h^2 + 3a + 3h - 1$

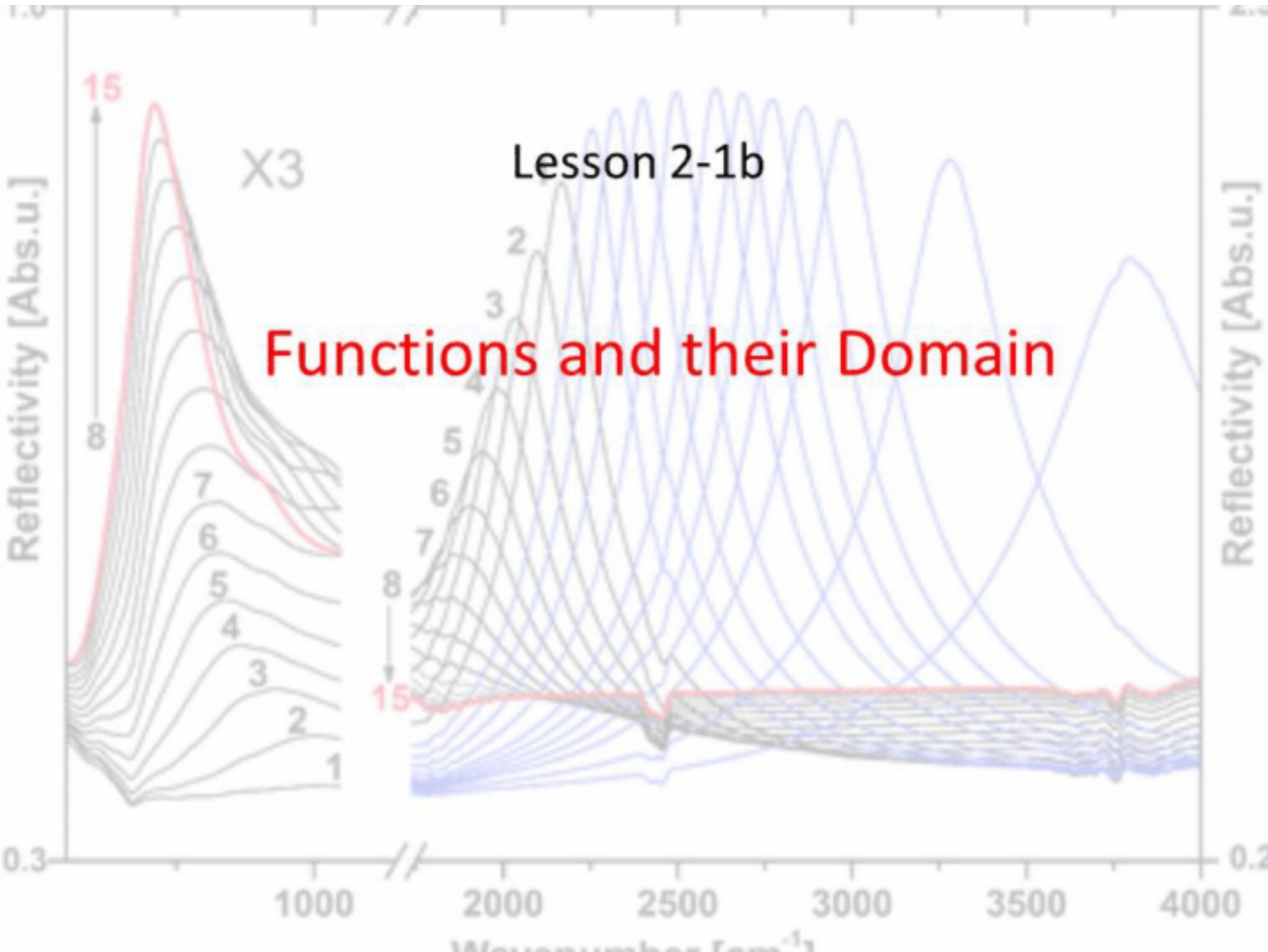
$$\frac{\cancel{2a^2} + 4ah + 2h^2 + \cancel{3a} + 3h \cancel{-1} \cancel{-2a^2} - \cancel{3a} \cancel{+1}}{h}$$

$$= \frac{4ah + 2h^2 + 3h}{h} = \frac{\cancel{h}(4a + 2h}{\cancel{h}}$$

$4a + 2h + 3$

Lesson 2-1b

Functions and their Domain



Objective

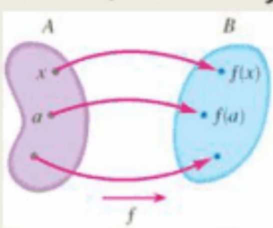
Students will…

- Be able to solve word problems using functional relationship.
- Be able to find the domains of functions.
- Be able to represent functions in multiple ways.

Definition of a Function

So now we are ready to define what a function is.

A **function**, say f, is a rule that assigns to each element (item) x in a certain set A **exactly one** element, called $f(x)$, in a set B.

Ex. 

Another way to define function is for every **input**, there is exactly **one** **output**.

The set A is also known as the **domain**, and set B is known as the **range**.

Word Problems Using Functions

If an astronaut weighs 130 pounds on the surface of the earth, then her weight when she is h miles above the earth is given by the function: $w(h) = 130\left(\frac{3960}{3960+h}\right)^2$

a. What is her weight when she is 100 mi above the earth?

$$w(100) = 130\left(\frac{3960}{3960+100}\right)^2 \approx 123.7 \text{ lbs.}$$

b. Construct a table of values of the function w that gives her weight at heights from 0 to 500 mi. What do you conclude from the table?

h	$w(h)$
100	
200	
300	
400	
500	

Word Problems Using Functions

If the speed limit on a 100-mile stretch of road is 75 miles per hour, then the amount of time it takes a car going x miles per hour over the limit to travel the stretch is given by $f(x) = \frac{100}{75+x}$

a. How long does it take the car to travel the stretch if the car is going 10 miles per hour over the limit?

$$f(10) = \frac{100}{75+10} = \frac{100}{85} \approx 1.17 \text{ hrs.}$$

b. How long does it take the car to travel the stretch if the car is not speeding at all?

Initial

$$f(0) = \frac{100}{75+0} = \frac{100}{75} = 80 \text{ min.}$$

Domain of a Function

Recall that the **domain** of a function is the set of all **inputs**. Domain may be written **explicitly**. For example, for the function $f(x) = x^2, \quad 0 \leq x \leq 5$, the domain is specifically set as all inputs between and including 0 and 5. Hence its domain is simply $[0, 5]$.

~~less~~ than, ~~not~~ equal to
greater " or equal to

Whenever we have a function without the domain stated explicitly, we need to figure it out by algebraic reasoning.

Ex. $f(x) = x^2 + 1$ $\qquad g(x) = \frac{1}{x-4}$ $\qquad h(x) = \sqrt{x}$

$(-\infty, \infty)$

$x - 4 = 0$

D: $x \neq 4$

$(-\infty, 4) \cup (4, \infty)$

$x \not< 0$

D: $x \geq 0$

$[0, \infty)$

$(-\infty, 0)$ $(0,1)$ $(1, \infty)$

Examples

$3 < 4$
$-3 > -4$

$\sqrt{9} = 3$
$-\sqrt{9} = -3$

Find the domain of each function.

a. $f(x) = \frac{1}{x^2 - x}$

$x^2 - x = 0$

$x(x-1) = 0$

D: $x \neq 0, 1$

$(-\infty, 0)$

"or" → $\cup$

$(0, 1)$

$\cup$

$(1, \infty)$

b. $g(x) = \sqrt{9 - x^2}$

$9 - x^2 < 0$

$+x^2 < +9$

$\sqrt{x^2} > \sqrt{9}$

D: $x \not> 3$, $x \not< -3$

D: $x \leq 3$, $x \geq -3$

$-3 \leq x \leq 3$

$[-3, 3]$

c. $h(t) = \frac{t}{\sqrt{t+1}}$

$t + 1 \leq 0$

D: $t \not\leq -1$

D: $t > -1$

$(-1, \infty)$

Four Ways of Representing a Function

To help us understand what a function is, we have used machine and arrow diagrams. We can represent a functional relationship in following ways:

1. Verbally (by a description in words)

2. Algebraically (by an explicit formula)

   $f(x) = x^2$

3. Visually (by a graph)

4. Numerically (by a table of values)

Four Ways to Represent a Function

Four Ways to Represent a Function

Verbal

Using words:

$P(t)$ is "the population of the world at time t"

Relation of population P and time t

Algebraic

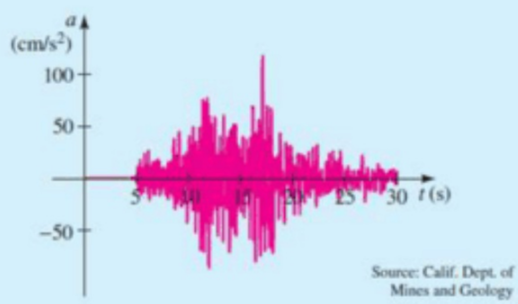
Using a formula:

$$A(r) = \pi r^2$$

Area of a circle

Visual

Using a graph:



Vertical acceleration during an earthquake

Numerical

Using a table of values:

w (ounces)	$C(w)$ (dollars)
$0 < w \le 1$	0.37
$1 < w \le 2$	0.60
$2 < w \le 3$	0.83
$3 < w \le 4$	1.06
$4 < w \le 5$	1.29
$\vdots$	$\vdots$

Cost of mailing a first-class letter

Homework 9/4

TB pg. 156 #34, 38, 42, 44, 50, 58, 59, 66