Lesson 2-2

Derivative II

Objective

Students will…

- Be able to know and use the basic differentiation rule.
- Be able to use the derivatives to find rates of change.
- Be able to relate derivative function to the velocity function.

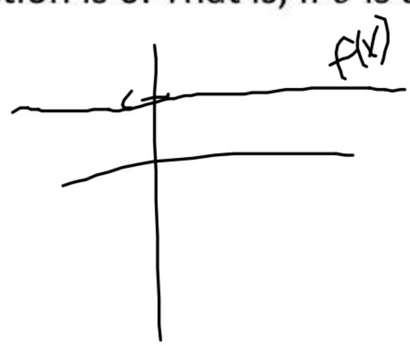
Constant Rule

You now know what differentiation is, and find derivative functions. As always in mathematics, it is always a good thing if certain things can be generalized into an easier or simpler form. The first and foremost, the most basic result of differentiation is none other than the **constant rule**.

The derivative of a constant function is 0. That is, if c is a real number, then

$$\frac{d}{dx}[c] = 0$$

f(x)=



Think the graph of any constant function. The slope is **ALWAYS** zero.

The Power Rule

The most important and useful rule in derivative would be the **power rule**.

The Power Rule- If n is a rational number, then the function $f(x) = x^n$ is differentiable and $\frac{d}{dx}[x^n] = nx^{n-1}$

For f to be differentiable at $x = 0$, n must be a number such that x^{n-1} is defined on an interval containing 0.

Proof of the Power Rule

$f(x) = x^n$

$f(x+h) = (x+h)^n$

$$f'(x) = \lim_{h \to 0}\left(\frac{(x+h)^n - x^n}{h} = \frac{\cancel{x^n} + x^{n-1}h + x^{n-2}h^2 + x^{n-3}h^3 + \cdots + h^n \cancel{- x^n}}{h}\right)$$

$$= \frac{\cancel{h}\left(x^{n-1} + x^{n-2}h + x^{n-3}h^2 + \cdots + h^{n-1}\right)}{\cancel{h}}$$

$$= x^{n-1}$$

Example

Find the derivative of the following:

a. $f(x) = x^3$

$= \boxed{3x^2}$

b. $g(x) = \sqrt[3]{x} = x^{1/3}$

$= \boxed{\frac{1}{3}x^{-2/3}}$

Example

Find the derivative of the following:

c. $y = \frac{1}{x^2} = x^{-2}$

$$\boxed{y' = -2x^{-3}}$$

Example

Find the equation of a tangent line to the graph of $f(x) = x^2$ when $x = -2$. (x, y) $(-2, 4)$

$f'(x) = 2x$

$f'(-2) = -4 = m$

$\boxed{y - 4 = -4(x+2)}$

Laws Derivatives

1. $\frac{d}{dx}[cf(x)] = cf'(x)$, where c is a real number.

$\underline{\text{ex.}}$ $3x^2 = 6x$ or $3x^2 = 3(x^2) = 3(2x) = 6x$.

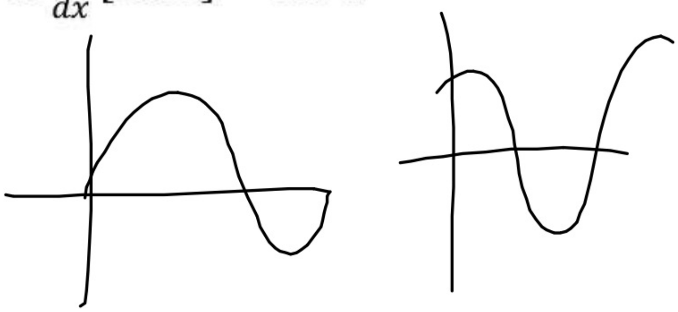
2. $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)] = f'(x) \pm g'(x)$

*<u>**NOTE**</u>: $\frac{d}{dx}[f(x) \times\div g(x)] \neq f'(x) \times\div g'(x)$

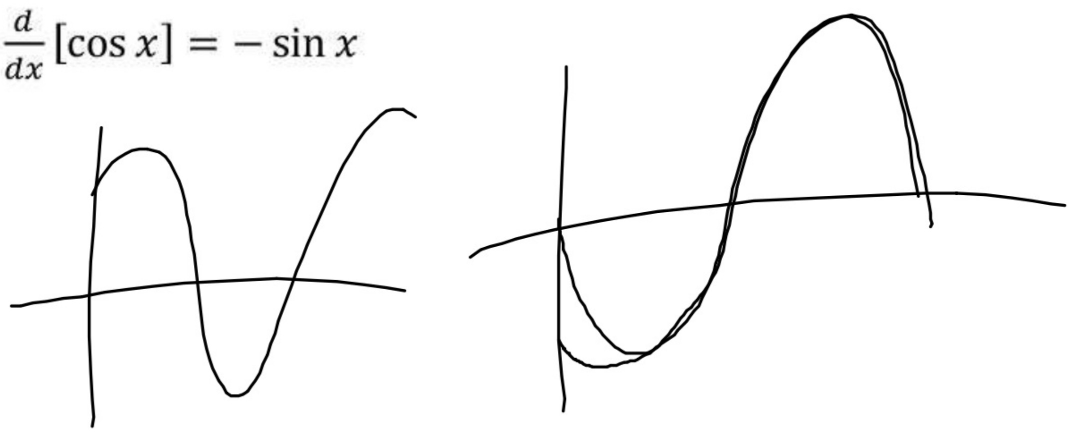
complementary to sine.

Trig Derivatives

1. $\frac{d}{dx}[\sin x] = \cos x$



2. $\frac{d}{dx}[\cos x] = -\sin x$



Avg vs Instantaneous Rate of Change

Ex. If a billiard ball is dropped from a height of 100 feet, its height s at time t is given by the position function: $s = -16t^2 + 100$, where s is measured in feet and t is measured in seconds. Find the average velocity (rate of change) over each of the following time intervals.

a. $[1, 2]$

a b

$$\frac{f(b)-f(a)}{b-a} = \frac{36-84}{1} = \frac{-48}{1} = \boxed{-48} \text{ ft/sec}$$

b. $[1, 1.5]$

a b

$$= \frac{64-84}{0.5} = \frac{-20}{0.5} = -40 \text{ ft/s.}$$

Avg vs Instantaneous Rate of Change

Ex. At time $t = 0$, a diver jumps from a platform diving board that is 32 feet above the water. The position of the diver is given by

$$s(t) = -16t^2 + 16t + 32,$$

where s is measured in feet and t is measured in seconds.

a. When does the diver hit the water?

$0 = -16t^2 + 16t + 32$

$0 = -16(t^2 - t - 2)$ $(t-2)(t+1)$ $\boxed{t=2}$, $\cancel{-1}$

b. What is the diver's velocity at impact?

$s'(t) = -32t + 16 + 0$ $s'(2) = -64 + 16 = \boxed{-48\ ft/s}$

Instantaneous Velocity. – Derivative.

Homework 9/27

Position function: original.

2.2 Exercises #3-23 (odd), 39-51 (odd), 93, 94, 103, 104