Lesson 1-4

Continuity

Objective

Students will…

- Be able to distinguish between removable and nonremovable discontinuities.
- Be able to define and use the intermediate value theorem.

Continuity

A function, say f, is **continuous at c** when ~~these~~ all three conditions are met:

"outputs"

1. $f(c)$ is defined. (i.e. can be evaluated).
2. $\lim_{x \to c} f(x)$ exists. (can be found)
3. $\lim_{x \to c} f(x) = f(c)$. (Direct substitution) "①" = "②"

Recall: We can show that a function has a limit at any given point by the existence of limit theorem:

$\lim_{x \to c^-} f(x) = L = \lim_{x \to c^+} f(x)$, (the right and the left side limits are equal)

Types of Discontinuity

Always remember that not all discontinuities are created equal! In fact, just because a discontinuity exists at a certain point, this doesn't automatically indicate that the limit doesn't exist. Consider the following problems:

a. $\lim\limits_{x\to 1}\left(\frac{x^2-1}{x-1} = \frac{(x+1)\cancel{(x-1)}}{\cancel{x-1}} = (x+1)\right)$

$= 1+1 = 2$

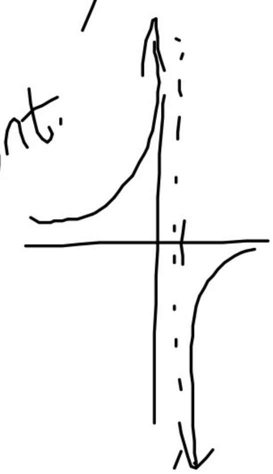
Removable Discont.

Limit exists

b. $\lim\limits_{x\to 1}\left(\frac{1}{x-1}\cdot\frac{x+1}{x+1} = \frac{x+1}{x^2-1}\right) = DNE$

Nonremovable Discont.

Limit DNE.



Types of Discontinuity

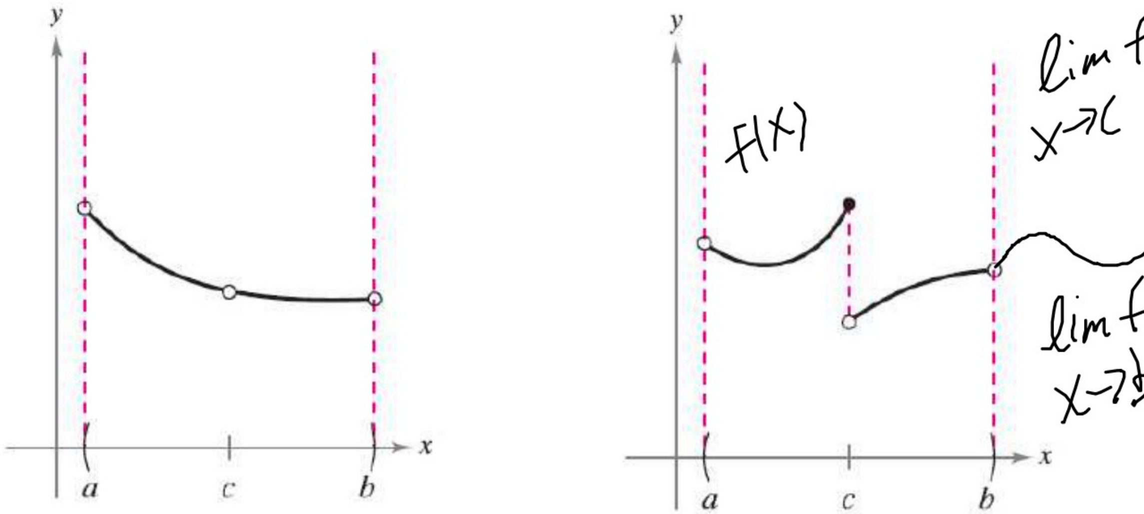
Clearly, (a) has a limit, while (b) did not. Algebraically speaking, simple factoring and simplifying allowed us to find the limit for (a), while there was nothing that could have been done for to find a limit for (b). This can be more easily seen looking at their graphs.

In general…

If the limit exists at a certain point of a function, say c, while the function is undefined at c, then the function is said to have a **removable discontinuity** at c

If the limit does not exist at c, nor is defined at c, then the function is said to have a **nonremovable discontinuity** at c.

Removable vs Nonremovable Discontinuity



(a) Removable discontinuity

(b) Nonremovable discontinuity

$\lim_{x \to c} f(x) = DNE$

$\lim_{x \to b} f(x) = \text{Exists.}$

Examples

Find points of discontinuity, and determine if they are removable, or nonremovable discontinuity (ies).

Limit Exist?

DNE?

a. $f(x) = \frac{4}{x-6}$

$x-6=0$

$x=6$

$\lim_{x \to 6} f(x) = DNE$

$x=6$ is a nonremovable discont. b/c

b. $f(x) = \frac{x-5}{x^2-25}$

$f(x) = \frac{\cancel{x-5}}{(x+5)\cancel{(x-5)}} = \frac{1}{x+5}$

$x^2-25=0$

$\sqrt{x^2} = \sqrt{25}$

$x = \pm 5$

$(x+5)(x-5)=0$

$\lim_{x \to 5} f(x) = \frac{1}{10} \Rightarrow$ removable disc. @ $x=5$

$\lim_{x \to -5} f(x) = DNE \Rightarrow$ nonremovable disc. @ $x=-5$

c. $f(x) = \frac{x+2}{x^2-x-6} = \frac{\cancel{x+2}}{\cancel{(x+2)}(x-3)} = \frac{1}{x-3}$

$x^2-x-6=0$

$(x+2)(x-3)=0$

$x=-2, 3$

$\lim_{x \to -2} f(x) = \frac{1}{-5}$

$\lim_{x \to 3} f(x) = DNE$

Intermediate Value Theorem

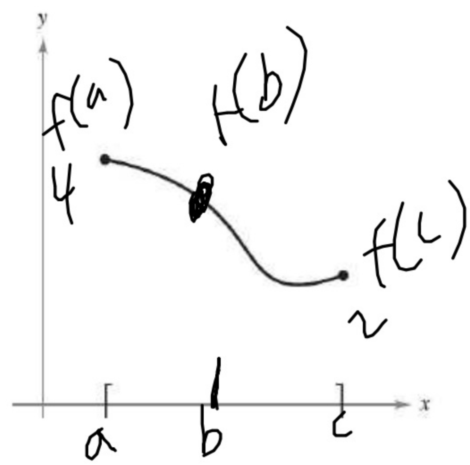
There is a very simple but important theorem in Calculus regarding continuity.

Intermediate Value Theorem- If f is continuous on the closed interval $[a, c]$, and $f(a) \neq f(c)$, and k is any number between $f(a)$ and $f(c)$, then there is at least one number b in $[a, c]$ such that $f(b) = k$.

c b is between a, c.

In other words, in the interval $[a, \cancel{b}]$, if $\cancel{a \leq b \leq c}$, then, $f(b)$ exists, such that $\cancel{f(a) \leq f(b) \leq f(c)}$.

f(b) is between f(a) & f(c)



Example

Use the Intermediate Value Theorem to show that the polynomial function $f(x) = x^3 + 2x - 1$ has a zero (x-intercept or root) in the interval [0,1].

$f(0) = 0^3 + 2(0) - 1 = -1$

$f(1) = 1^3 + 2(1) - 1 = 2$

By IVT, there must be c in $[0,1]$, such that $f(c) = 0$.

Example

Use the Intermediate Value Theorem to show that the function

$f(x) = -\frac{5}{x} + \tan\frac{\pi x}{10}$ has a zero (x-intercept or root) in the interval $[1,4]$.

$f(1) = -\frac{5}{1} + \tan\frac{\pi}{10} \approx -0.15$

$f(4) = -\frac{5}{4} + \tan\frac{4\pi}{10} \approx 1.83$

By IVT ~~that~~ certain c exists in $[1,4]$ such that $f(c)=0$

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Homework 9/11

1.4 exercises #35-47 (odd), 48, 51, 53, 83-84