Experiment - a process that gives definite results.

ex. tossing a coin, rolling a die.

Outcome - the definite results of an experiment.

Sample Space (S) — all the possible outcomes of an experi

ex. Tossing a coin. $S=\{H,T\}$

Rolling a die $S=\{1,2,3,4,5,6\}$.

Event - any subset of the Sample space. Denoted E.

$\hookrightarrow$ set within a set.

ex. Rolling a die. Event of rolling an odd number;

$S=\{1,2,3,4,5,6\}$., $E=\{1,3,5\}$.

Ex 1. For an experiment of tossing a coin 3 times, What is the Sample Space?

$$S = \{HHH, HHT, HTH, THH, HTT, TTH, THT, TTT\}.$$

How many outcomes exist for the event of getting exactly 2 Heads?

3 outcomes – HHT, HTH, THH.

How many outcomes exist for the event of getting NO heads?

1 outcome – TTT.

Definition of Probability – Let S be the sample space, in which all outcomes are equally likely, and let E be an event. The probability of E, written $P(E)$, is

$$P(E) = \frac{\#\text{ of outcomes for } E}{\text{total }\#\text{ of outcomes in } S} = \frac{n(E)}{n(S)} \qquad \frac{\text{Part}}{\text{whole}}$$

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Ex. 1 $S = \{HHH, HHT, HTH, THH, TTH, HTT, THT, TTT\}$.

$$P(\text{getting exactly 2 heads}) = \frac{n(E)}{n(S)} = \frac{3}{8}$$

$$P(\text{getting at least 2 heads}) = \frac{n(E)}{n(S)} = \frac{4}{8}$$

$$P(\text{getting no heads}) = \frac{n(E)}{n(S)} = \frac{1}{8}$$

$$\frac{\text{Part}}{\text{whole}}$$

Ex. 2. A five-card Poker hand is drawn from a standard 52-card deck. What is the probability that all five cards are spades?

Sample Space: $C(52,5) = \frac{52!}{5!(52-5)!} = 2,598,960$

Event: $C(13,5) = \frac{13!}{5!(13-5)!} = 1287$

$$P(E) = \frac{n(E)}{n(S)} = \frac{1287}{2,598,960} \approx \boxed{0.0005}$$

E = drawing 5 odd numbers

event $= C(16,5) = \frac{16!}{5!(16-5)!} = 4368$

$$P(E) = \frac{n(E)}{n(S)} = \frac{4368}{2,598,960} \approx \boxed{0.002}$$

Complement of an event – a set of outcomes in the sample space that is not in E.

Probability of the complement – Let E be an event. Then, the probability of its complement, denoted, E,′ is,

$$P(E') = 1 - P(E).$$

$P(E') = 1 - P(E)$
$+P(E) \quad +P(E)$

$P(E') + P(E) = 1$
$-P(E') \quad -P(E')$

$P(E) = 1 - P(E')$.

Ex. Rolling a die.

$P(3) = \frac{1}{6}$

$P(3') = 1 - \frac{1}{6} = \frac{6}{6} - \frac{1}{6} = \frac{5}{6}$

Ex. 6 An urn contains 10 red balls and 15 blue balls. 6 balls drawn at random. What is the probability of drawing at least 1 red ball?

E = drawing at least 1 red ball.

E' = drawing no red balls.

$$P(E') = \frac{n(E')}{n(S)} = \frac{5005}{177100} = \frac{13}{460} \approx 0.03$$

$$n(S) = C(25,6) = \frac{25!}{6!(25-6)!} = 177100$$

$$n(E') = (15,6) = \frac{15!}{6!(15-6)!} = 5005$$

$$P(E) = 1 - P(E') = 1 - \frac{13}{460} = \frac{460}{460} - \frac{13}{460} = \frac{447}{460} \approx \boxed{0.97}$$

"Or/And" Probability.

"Or"

Mutually Exclusive Events - Two or more events with no common outcomes.

Probability of mutually exclusive events?

- Let E and F be mutually exclusive events. Then, the

$$P(E \overset{\text{"or"}}{\cup} F) = P(E) + P(F)$$

ex. E = rolling a 3

$P(E) = 1/3$ $P(F) = 3/6$

F = rolling an even #

$$P(E \cup F) = P(E) + P(F) = \frac{1}{6} + \frac{3}{6} = \boxed{4/6} \approx \boxed{.667}$$

"And"

Independence – Two or more events that do not affect each others' probability.

drawing 2 cards w/ replacement. drawing w/o replacement.

ex. P(A and 4) vs. P(A and 4).

Probability of two independent events.

- Let E and F be independent events.

$$P(E \overset{\text{"and"}}{\cap} F) = P(E) \cdot P(F)$$

ex1.

$$P(A \text{ and } \heartsuit) = P(A) \cdot P(\heartsuit) = \frac{\cancel{4}^{1}}{\cancel{52}_{4}} \cdot \frac{\cancel{13}^{1}}{52} = 1 \cdot \frac{1}{52} = \boxed{\frac{1}{52}}$$