

Day 2 Permutation and Combination.

ex. 1 Consider the following 8 letters: ABCD WXYZ. How many different ways can these letters be arranged?

$$\underline{8} \quad \underline{7} \quad \underline{6} \quad \underline{5} \quad \underline{4} \quad \underline{3} \quad \underline{2} \quad \underline{1}$$

By Fundamental Counting Principle, $: 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
 $= 8! = \boxed{40,320}$

The number of permutation for n number of "items" is

$$\boxed{n!}$$

ex. 2 From the 8 letters in ex. 1, how many ways can you arrange 5 of those letters?

$$\underline{8} \quad \underline{7} \quad \underline{6} \quad \underline{5} \quad \underline{4} = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = \boxed{6720}$$
$$= 8 \cdot (8-1) \cdot (8-2) \cdot (8-3) \cdot (8-4) = 6720$$

The number of permutation for n number of "items" is $n!$

$P(8,5)$:

$$\underline{8} \underline{7} \underline{6} \underline{5} \underline{4} = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = 6720$$

$$= 8 \cdot (8-1) \cdot (8-2) \cdot (8-3) \cdot (8-4) = 6720$$

$P(n,r)$ n, r are numbers, $n > r$.

$$P(n,r) = \frac{n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdots (n-(r-1))}{(n-r)(n-r+1) \cdots 3 \cdot 2 \cdot 1}$$

$$P(n,r) = \frac{n!}{(n-r)!}$$

Ex. 2 From the 8 letters in ex. 1, how many ways can you arrange 5 of those letters?

ex. 3 A club has 9 members. In how many ways can a president, Vice-president, and secretary be chosen?

$$P(9, 3) = \frac{9!}{(9-3)!} = \frac{9!}{6!} = \frac{9 \cdot 8 \cdot 7 \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} = \boxed{504}$$

ex. 4. Out of 31 movie posters, how many different arrangement can you make to display 20 posters?

$$P(31, 20) = \frac{31!}{(31-20)!}$$

Combination

$$C(n, r) = \frac{n!}{r!(n-r)!}$$

ex 5 Juanya is planning a party. He has 9 friends. Now, due to budget limitations, he can only invite 5 of his 9 friends. Considering the fact that Juanya is a "happy-go-lucky" ^{50%} "love-everyone-equally" dude, how many different ways can he invite the 5 friends?

$$C(9, 5) = \frac{9!}{5!(9-5)!} = \frac{9!}{5!4!} = \frac{9 \times 8 \times 7 \times 6 \times 5 \times \cancel{4 \times 3 \times 2 \times 1}}{(\cancel{5 \times 4 \times 3 \times 2 \times 1})(4 \times 3 \times 2 \times 1)}$$
$$= \frac{3024}{24} = \boxed{126}$$