Warm Up 5/6

Find the determinant of the following:

$$B = \begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix}$$

$|B| = 14 - 15 = \boxed{-1}$

$$\begin{bmatrix} 4 & 5 \\ 0 & -1 \end{bmatrix}$$

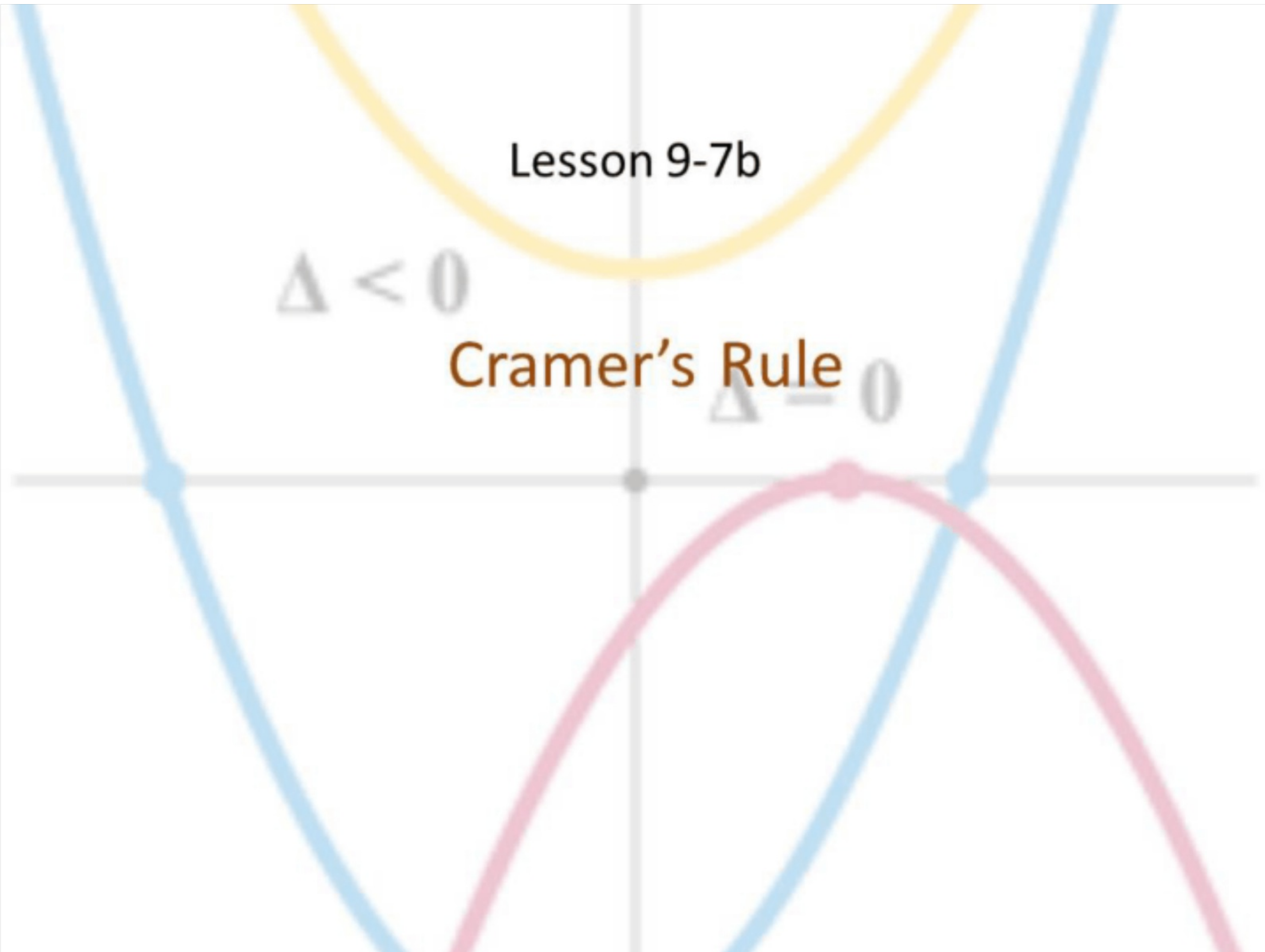
$\det = -4 - 0 = \boxed{-4}$

$$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

$\det = \boxed{6}$

Lesson 9-7b

Cramer's Rule



Objective

Students will…

- Be able to **correctly** (shame on you Lee) find the determinant of $n \times n$ matrices.
- Be able to use Cramer's Rule to solve system of linear equations.

Determinants of any $n \times n$ Matrices

Now that we can always easily compute the determinant of any 2×2 matrix, let's see how we can compute the determinants of all other $n \times n$ square matrices. The idea is to pick a row or a column and split them up into smaller matrices. This is called **expanding by row or column**. Also, while doing this process, the signs will always alternate. The **top left-most entry will always be positive**.

Ex. Evaluate the determinant of the matrix,

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 0 & 2 & 4 \\ -2 & 5 & 6 \end{bmatrix} \Rightarrow |A| = 2\left|\begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}\right| - 3\left|\begin{bmatrix} 0 & 4 \\ -2 & 6 \end{bmatrix}\right| + -1\left|\begin{bmatrix} 0 & 2 \\ -2 & 5 \end{bmatrix}\right|$$

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$= 2(-8) - 3(8) + -1(4)$$

$$= -16 - 24 - 4 = \boxed{-44}$$

Picking a Row or Column for Expansion

We can pick **<u>any</u>** row or column to achieve this, but for ease, it is always wise and quicker if we choose the row or column with the most number of **<u>zero entries</u>**. Consider the previous example again. $\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$

Ex. Evaluate the determinant of the matrix,

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 0 & 2 & 4 \\ -2 & 5 & 6 \end{bmatrix} \Rightarrow |A| = \cancel{-0\left|\begin{bmatrix} 3 & -1 \\ 5 & 6 \end{bmatrix}\right|} + 2\left|\begin{bmatrix} 2 & -1 \\ -2 & 6 \end{bmatrix}\right| - 4\left|\begin{bmatrix} 2 & 3 \\ -2 & 5 \end{bmatrix}\right|.$$

$$= 2(10) - 4(16) = \boxed{-44}$$

Cramer's Rule

One useful application of determinants come from the fact that we can apply **Cramer's Rule** in order to solve system of linear equations.

~~augmented matrix~~

Cramer's Rule for Systems in Two Variables

The linear system

$$\begin{cases} ax + by = r \\ cx + dy = s \end{cases}$$

has the solution

$$x = \frac{\begin{vmatrix} r & b \\ s & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} a & r \\ c & s \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$$

provided $\begin{vmatrix} a & b \\ c & d \end{vmatrix} \neq 0$.

Example

Use Cramer's Rule to solve the system.

$$\begin{bmatrix} 2 & 6 \\ 1 & 8 \end{bmatrix} \leftarrow \begin{cases} 2x + 6y = -1 \\ x + 8y = 2 \end{cases} \rightarrow \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

denom: $\left|\begin{bmatrix} 2 & 6 \\ 1 & 8 \end{bmatrix}\right| = \boxed{10}$

$$x = \frac{\left|\begin{bmatrix} -1 & 6 \\ 2 & 8 \end{bmatrix}\right|}{10}, \quad y = \frac{\left|\begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}\right|}{10}$$

$$x = \frac{-20}{10} = \boxed{-2} \qquad y = \frac{5}{10} = \boxed{\frac{1}{2}}$$

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}.$$

Example

Use Cramer's Rule to solve the system.

$$\begin{cases} 2x - 3y + 4z = 1 \\ x \qquad\; + 6z = 0 \\ 3x - 2y \qquad = 5 \end{cases}$$

denom: $\left|\begin{bmatrix} 2 & -3 & 4 \\ 1 & 0 & 6 \\ 3 & -2 & 0 \end{bmatrix}\right|$

$= 4\left|\begin{bmatrix} 1 & 0 \\ 3 & -2 \end{bmatrix}\right| - 6\left|\begin{bmatrix} 2 & -3 \\ 3 & -2 \end{bmatrix}\right|$

$= 4(-2) - 6(5) = \boxed{-38}$

$x = \dfrac{\left|\begin{bmatrix} 1 & -3 & 4 \\ 0 & 0 & 6 \\ 5 & -2 & 0 \end{bmatrix}\right|}{-38}$, $y = \dfrac{\left|\begin{bmatrix} 2 & 1 & 4 \\ 1 & 0 & 6 \\ 3 & 5 & 0 \end{bmatrix}\right|}{-38}$, $z = \dfrac{\left|\begin{bmatrix} 2 & -3 & 1 \\ 1 & 0 & 0 \\ 3 & -2 & 5 \end{bmatrix}\right|}{-38}$.

$x = \dfrac{-6\left|\begin{bmatrix} 1 & -3 \\ 5 & -2 \end{bmatrix}\right|}{-38} = \dfrac{-6(13)}{-38} = \boxed{\dfrac{39}{19}}$

$= \dfrac{-1\left|\begin{bmatrix} 1 & 4 \\ 5 & 0 \end{bmatrix}\right| - 6\left|\begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}\right|}{-38}$ $\boxed{\dfrac{22}{38}} = \boxed{\dfrac{11}{19}}$

$= \dfrac{-1\left|\begin{bmatrix} -3 & 1 \\ -2 & 5 \end{bmatrix}\right|}{-38} = \dfrac{-1(-13)}{-38}.$

$z = \boxed{\dfrac{-13}{38}}.$

Homework Problem

Use Cramer's Rule to solve the system.

35. $$\begin{cases} x - y + 2z = 0 \\ 3x + z = 11 \\ -x + 2y = 0 \end{cases}$$

Homework 5/6

TB pg. 713 #29-41 (odd)