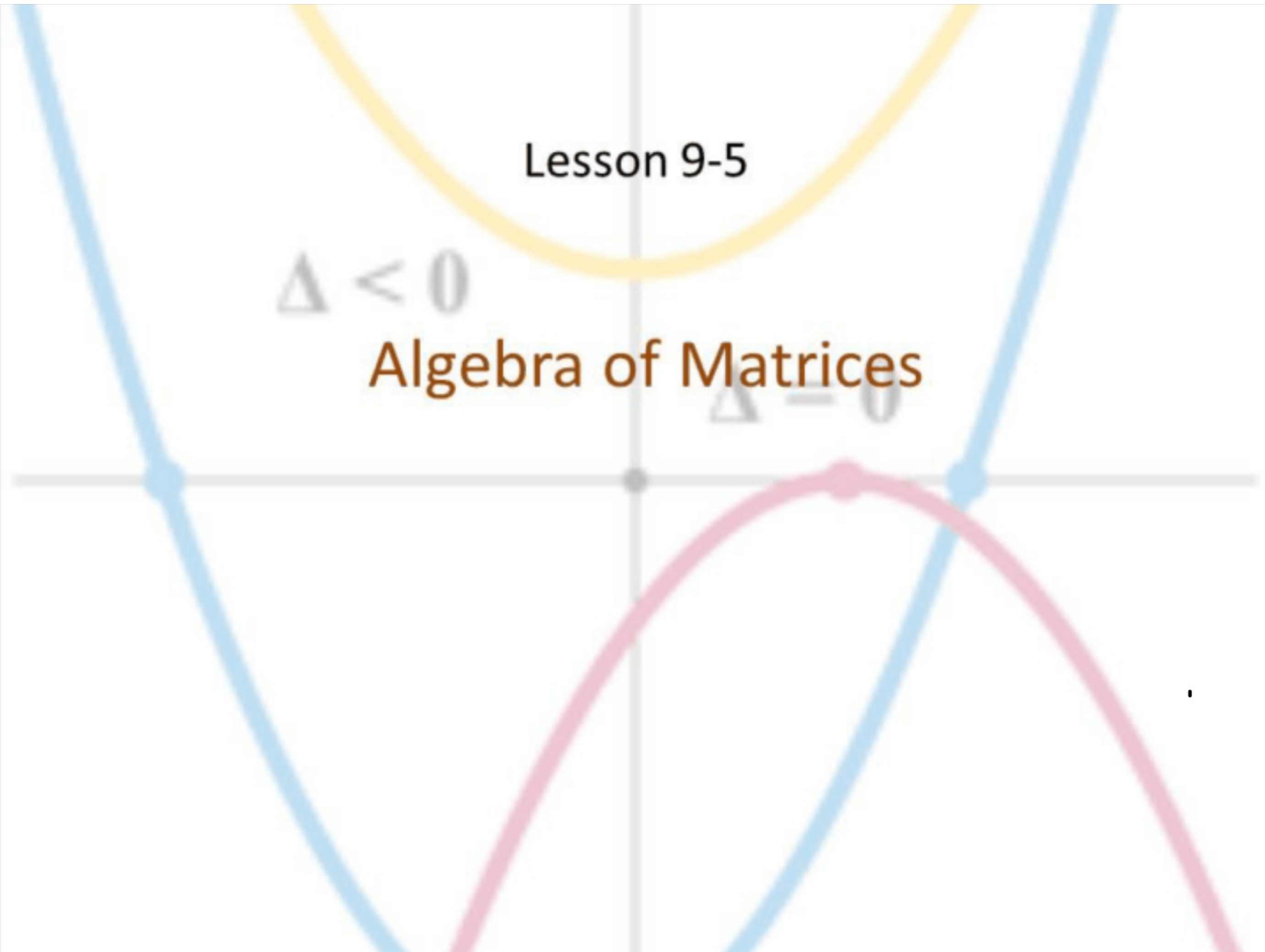
Lesson 9-5

Algebra of Matrices



Objective

Students will…

- Be able to add, subtract, and multiply matrices.
- Be able to know and apply the algebraic properties of matrices.

Matrices

In mathematics, a **matrix** is a **rectangular array** of numbers with rows and columns.

Ex. $\begin{bmatrix} 3 & -2 & 1 & 5 \\ 1 & 3 & -1 & 0 \\ -1 & 0 & 4 & 11 \end{bmatrix}$

Every matrix has a dimension, which will always be written in the form, $\boldsymbol{row \times column}$. In other words, an $m \times n$ matrix is a matrix with m rows and n columns.

Ex.

Matrix	Dimension
$\begin{bmatrix} 1 & 3 & 0 \\ 2 & 4 & -1 \end{bmatrix}$	2×3
$\begin{bmatrix} 6 & -5 & 0 & 1 \end{bmatrix}$	1×4

Example

Identify the dimensions of each matrix.

$A = \begin{bmatrix} 2 & -5 \\ 0 & 7 \end{bmatrix}$ $B = \begin{bmatrix} 3 & \frac{1}{2} & 5 \\ 1 & -1 & 3 \end{bmatrix}$ $C = \begin{bmatrix} 2 & -\frac{5}{2} & 0 \\ 0 & 2 & -3 \end{bmatrix}$

$D = \begin{bmatrix} 7 & 3 \end{bmatrix}$ $E = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$

A = 2x2 B = 2x3 C = 2x3

D = 1X2 E = 3x1

Matrix Addition and Subtraction

Basic operations, such as addition, subtraction, and multiplication can be done using matrices. We will first look at addition and subtraction.

Addition and subtraction between matrices can simply be done by either adding or subtracting each of the **corresponding entries**.

$$\begin{bmatrix} a & b \\ 1 & b \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 0 \\ 4 & 0 \\ 2 & 0 \end{bmatrix}$$

Ex. Let

$$A = \begin{bmatrix} 2 & -3 \\ 0 & 5 \\ 7 & -\frac{1}{2} \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 \\ -3 & 1 \\ 2 & 2 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 2+1 & -3+0 \\ 0+-3 & 5+1 \\ 7+2 & -\frac{1}{2}+2 \end{bmatrix} = \begin{bmatrix} 3 & -3 \\ -3 & 6 \\ 9 & 3/2 \end{bmatrix} \qquad A - B = \begin{bmatrix} 1 & -3 \\ 3 & 4 \\ 5 & -\frac{5}{2} \end{bmatrix}$$

Note: This is very similar to **vector addition and subtraction**.

Scalar Multiplication on Matrices

We can also multiply matrices by a **scalar**. Multiplying matrices by a scalar is very similar to scalar multiplication on **vectors**.

Ex. Let $A = \begin{bmatrix} 2 & -3 \\ 0 & 5 \\ 7 & -\frac{1}{2} \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 \\ -3 & 1 \\ 2 & 2 \end{bmatrix}$

$5A = 5\begin{bmatrix} 2 & -3 \\ 0 & 5 \\ 7 & -1/2 \end{bmatrix} = \begin{bmatrix} 10 & -15 \\ 0 & 25 \\ 35 & -\frac{5}{2} \end{bmatrix}$ $11B = 11\begin{bmatrix} 1 & 0 \\ -3 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 11 & 0 \\ -33 & 11 \\ 22 & 22 \end{bmatrix}$

Properties of Matrix Add/Subt, and Scalar

The properties of matrix addition, subtraction, and scalar multiplication are very similar to real-number properties.

Properties of Addition and Scalar Multiplication of Matrices

Let A, B, and C be $m \times n$ matrices and let c and d be scalars.

$A + B = B + A$	Commutative Property of Matrix Addition
$(A + B) + C = A + (B + C)$	Associative Property of Matrix Addition
$c(dA) = (cd)A$ $\quad 3(5A) = 5(3A)$ $(3 \cdot 5)A$	Associative Property of Scalar Multiplication
$(c + d)A = cA + dA$	Distributive Properties of Scalar
$c(A + B) = cA + cB$	Multiplication

Matrix Multiplication (Dot Product).

Multiplication on matrices are a bit more difficult. The following components are very important.

For any two matrices, say A and B,

- $AB \neq BA$.

- AB is only defined when the number columns in A matches the number of rows in B.

$$\underset{2\times 2}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}} \cdot \underset{2\times 3}{\begin{bmatrix} 1 & 2 & 5 \\ 0 & 3 & 7 \end{bmatrix}} =$$

- The resulting product of AB will have the dimension of the number of rows in A by the number of columns in B.

  Ex. If A is $m \times n$ and B is $n \times k$, then AB is $m \times k$

Matrix Multiplication

With that said, matrix multiplication, say AB, is done by taking each of the **rows in A** and **multiplying** it to each of the **columns in B**. (Dot Product)

Ex. Let

$$A = \begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -1 & 5 & 2 \\ 0 & 4 & 7 \end{bmatrix}$$

$$2 \times 2 \qquad 2 \times 3$$

$$AB = \begin{bmatrix} 1\cdot -1 + 3\cdot 0 & 1\cdot 5 + 3\cdot 4 & 1\cdot 2 + 3\cdot 7 \\ -1\cdot -1 + 0\cdot 0 & -1\cdot 5 + 0\cdot 4 & -1\cdot 2 + 0\cdot 7 \end{bmatrix} = \begin{bmatrix} -1 & 17 & 23 \\ 1 & -5 & -2 \end{bmatrix}$$

$$BA = \cancel{2 \times 3 \quad 2 \times 2}$$

Note that AB has dimension 2×3.

Examples

$$\begin{bmatrix} 2 & 6 \\ 1 & 3 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 3 & 6 \\ -2 & 0 \end{bmatrix} = \text{undefined}$$

$$3\times\cancel{2} \qquad \cancel{3}\times 2$$

$$\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 1 & -2 & 3 \\ 2 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 1\cdot 1 + 2\cdot 2 & 1\cdot -2 + 2\cdot 2 & 1\cdot 3 + 2\cdot -1 \\ -1\cdot 1 + 4\cdot 2 & -1\cdot -2 + 4\cdot 2 & -1\cdot 3 + 4\cdot -1 \end{bmatrix} = \begin{bmatrix} 5 & 2 & 1 \\ 7 & 10 & -7 \end{bmatrix}$$

$$2\times 2 \qquad 2\times 3$$

$$\begin{bmatrix} 2 & 1 & 2 \\ 6 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 3 & 6 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 2\cdot 1 + 1\cdot 3 + 2\cdot -2 & 2\cdot 2 + 1\cdot 6 + 2\cdot 0 \\ 6\cdot 1 + 3\cdot 3 + 4\cdot -2 & 6\cdot -2 + 3\cdot 6 + 4\cdot 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 7 & 6 \end{bmatrix}$$

$$2\times 3 \qquad 3\times 2$$

Properties of Matrix Multiplication

Matrix multiplication follows some of the real number properties as well.

Properties of Matrix Multiplication

Let A, B, and C be matrices for which the following products are defined. Then

$A(BC) = (AB)C$	Associative Property
$A(B + C) = AB + AC$	Distributive Property
$(B + C)A = BA + CA$	

Homework 5/1

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