Warm Up 4/22

Solve the following system of equation

$$\begin{cases} x - 2y - 3z = 5 \\ 2x + y - z = 5 \\ 4x - 3y - 7z = 5 \end{cases}$$

$-12 - -7$

$-8 - -3$

$-6 - -1$

$2(x - 2y - 3z) = 5(2)$

$2x + y - z = 5$

$2x - 4y - 6z = 10$

$\ominus\ 2x + y - z = 5$

$-5y - 5z = 5$

$4(x - 2y - 3z) = 5(4)$

$4x - 3y - 7z = 5$

$4x - 8y - 12z = 20$

$\ominus\ 4x - 3y - 7z = 5$

$-5y - 5z = 15$

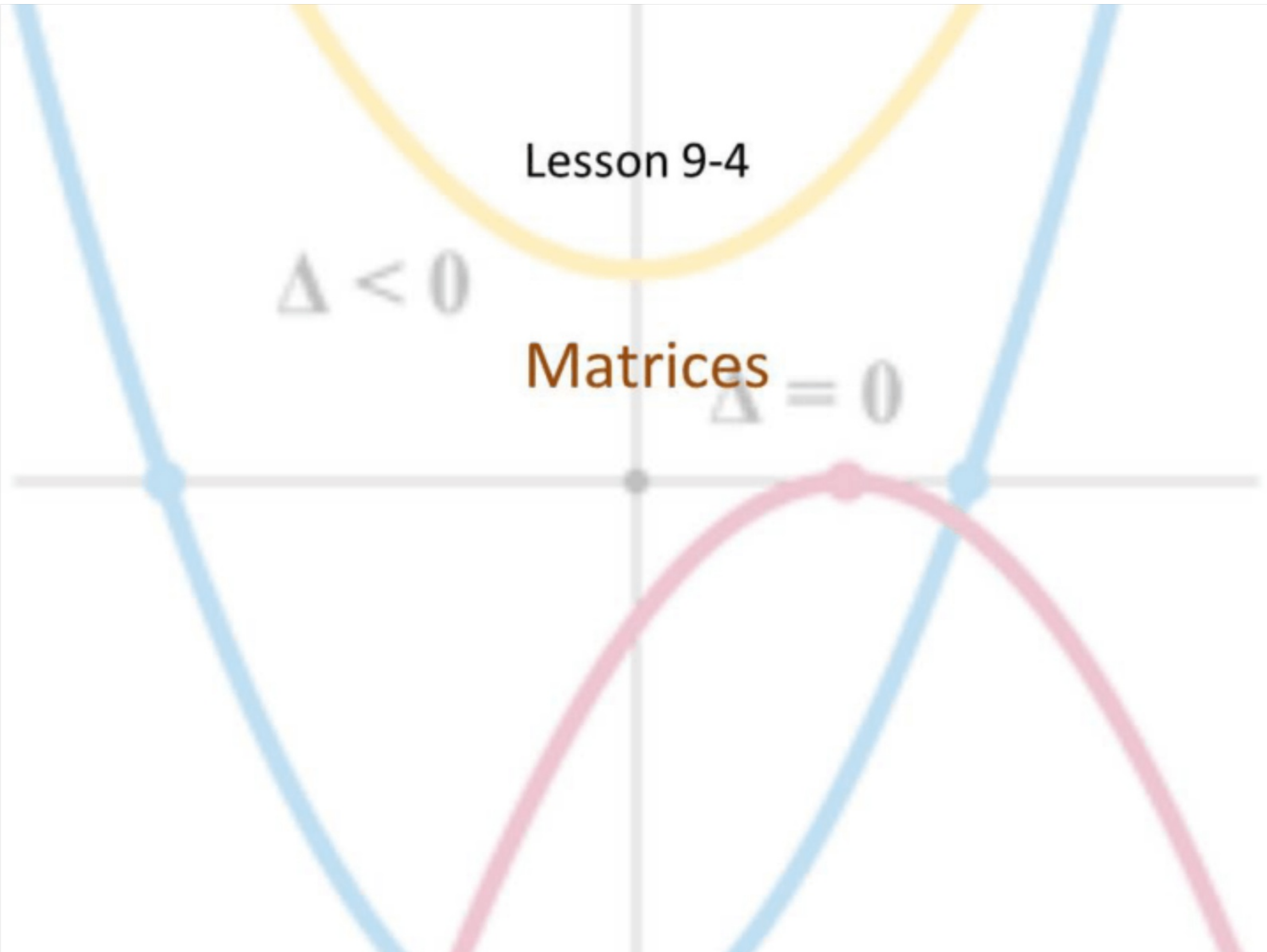
$0 = 0 \checkmark$

$5 \overset{?}{\neq} 15$

No Real Solution.

Lesson 9-4

Matrices



Objective

Students will…

- Be able to define a matrix and know its dimension.
- Be able to perform elementary row operation on any given matrix in order to turn it into row-echelon form.

Matrices

In mathematics, a **matrix** is a **rectangular array** of numbers with rows and columns.

Ex.

$$\begin{bmatrix} 3 & -2 & 1 & 5 \\ 1 & 3 & -1 & 0 \\ -1 & 0 & 4 & 11 \end{bmatrix}$$

Every matrix has a dimension, which will always be written in the form, $\boldsymbol{row \times column}$. In other words, an $m \times n$ matrix is a matrix with m rows and n columns.

Ex.

Matrix	**Dimension**
$\begin{bmatrix} 1 & 3 & 0 \\ 2 & 4 & -1 \end{bmatrix}$	2×3
$\begin{bmatrix} 6 & -5 & 0 & 1 \end{bmatrix}$	1×4

Elementary Row Operations

There are certain ways a matrix can be "safely" modified. These are known as **elementary row operations**. They are given as follows.

Elementary Row Operations

1. Add a multiple of one row to another
   Symbol: $\cancel{R_i + kR_j \to R_i}$ $kR_j + \boxed{R_i}$ Changed.
2. Multiply a row by a nonzero constant
   Symbol: kR_i
3. Interchange two rows.
   Symbol: $R_i \leftrightarrow R_j$

These three operations can **always be done** in **any matrix**. The usefulness of these operations will be explored in the next section. For now, our goal is to get used to doing these operations on matrices.

Example

Let's try these operations on an actual matrix!

$$\begin{bmatrix} 1 & -1 & 3 & 4 \\ 1 & 2 & -2 & 10 \\ 3 & -1 & 5 & 14 \end{bmatrix}$$

1. Add a multiple of one row to another

$$\begin{bmatrix} 1 & -1 & 3 & 4 \\ 1 & 2 & -2 & 10 \\ 3 & -1 & 5 & 14 \end{bmatrix} \xrightarrow{-1R_1 + R_2} \begin{bmatrix} 1 & -1 & 3 & 4 \\ 0 & 3 & -5 & 6 \\ 3 & -1 & 5 & 14 \end{bmatrix}$$

2. Multiply a row by a nonzero constant

$$\begin{bmatrix} 1 & -1 & 3 & 4 \\ 1 & 2 & -2 & 10 \\ 3 & -1 & 5 & 14 \end{bmatrix} \xrightarrow{-3R_3} \begin{bmatrix} 1 & -1 & 3 & 4 \\ 1 & 2 & -2 & 10 \\ -9 & 3 & -15 & -42 \end{bmatrix}$$

3. Interchange two rows.

$$\begin{bmatrix} 1 & -1 & 3 & 4 \\ 1 & 2 & -2 & 10 \\ 3 & -1 & 5 & 14 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_3} \begin{bmatrix} 3 & -1 & 5 & 14 \\ 1 & 2 & -2 & 10 \\ 1 & -1 & 3 & 4 \end{bmatrix}$$

Row-Echelon Form

The main reason for these operations is so that we can rewrite the matrix in what's known as, **Row-Echelon Form**. A matrix is in Row-Echelon Form if it satisfies the following conditions:

1. The first nonzero number in each row (reading from left to right) is 1. This is called the **leading entry**.
2. The leading entry in each row is to the right of the leading entry in the row immediately above it.
3. All rows consisting entirely of zeroes are at the bottom of the matrix.

Ex.

$$\begin{bmatrix} \boxed{1} & 3 & -6 & 10 & 0 \\ 0 & 0 & 1 & 4 & -3 \\ 0 & 0 & 0 & \boxed{1} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Row-Echelon Form

Here is a systematic guideline for putting a matrix into row-echelon form.

- Start by obtaining 1 in the top left corner. Then obtain zeros below that 1 by adding appropriate multiples of the first row to the rows below it.
- Next, obtain a leading 1 in the next row, and then obtain zeros below that 1.
- At each stage make sure that every leading entry is to the right of the leading entry in the row above it—rearrange the rows if necessary.
- Continue this process until you arrive at a matrix in row-echelon form.

We will see just how powerful matrices in row-echelon form can be in the next lesson.

Example

Let's put a matrix into row-echelon form.

$$\begin{bmatrix} 1 & -1 & 3 & 4 \\ 1 & 2 & -2 & 10 \\ 3 & -1 & 5 & 14 \end{bmatrix} \xrightarrow[-3R_1+R_3]{-1R_1+R_2} \begin{bmatrix} 1 & -1 & 3 & 4 \\ 0 & 3 & -5 & 6 \\ 0 & 2 & -4 & 2 \end{bmatrix}$$

$$\xrightarrow{-1R_3+R_2} \begin{bmatrix} 1 & -1 & 3 & 4 \\ 0 & 1 & -1 & 4 \\ 0 & 2 & -4 & 2 \end{bmatrix} \xrightarrow{-2R_2+R_3} \begin{bmatrix} 1 & -1 & 3 & 4 \\ 0 & 1 & -1 & 4 \\ 0 & 0 & -2 & -6 \end{bmatrix}$$

$$\xrightarrow{-\frac{1}{2}R_3} \begin{bmatrix} 1 & -1 & 3 & 4 \\ 0 & 1 & -1 & 4 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

You try...

Write the following matrix into row-echelon form.

$$\begin{bmatrix} 1 & 2 & -3 & -4 & 10 \\ 1 & 3 & -3 & -4 & 15 \\ 2 & 2 & -6 & -8 & 10 \end{bmatrix} \xrightarrow[-2R_1+R_3]{-1R_1+R_2} \begin{bmatrix} 1 & 2 & -3 & -4 & 10 \\ 0 & 1 & 0 & 0 & 5 \\ 0 & -2 & 0 & 0 & -10 \end{bmatrix}.$$

$$\xrightarrow{-2R_2+R_3} \begin{bmatrix} 1 & 2 & -3 & -4 & 10 \\ 0 & 1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Homework 4/22

Row-Echelon Form WKSHT