# Warm Up 4/10

Convert the following into polar form.

$x = r\cos\theta \quad y = r\sin\theta$

1. $x + y = 4$

$r\cos\theta + r\sin\theta = 4$

$\frac{r(\cos\theta + \sin\theta)}{\cos\theta + \sin\theta} = \frac{4}{\cos\theta + \sin\theta}$

$\boxed{r = \frac{4}{\cos\theta + \sin\theta}}$

2. $(x^2 + y^2)^2 = 2xy$

$((r\cos\theta)^2 + (r\sin\theta)^2)^2 = 2(r\cos\theta)(r\sin\theta)$

$= (r^2\cos^2\theta + r^2\sin^2\theta)^2 = 2r^2\cos\theta\sin\theta$

$= (r^2(\underbrace{\cos^2\theta + \sin^2\theta}_{1}))^2 = \text{''}$

$= (r^2)^2 = \text{''}$

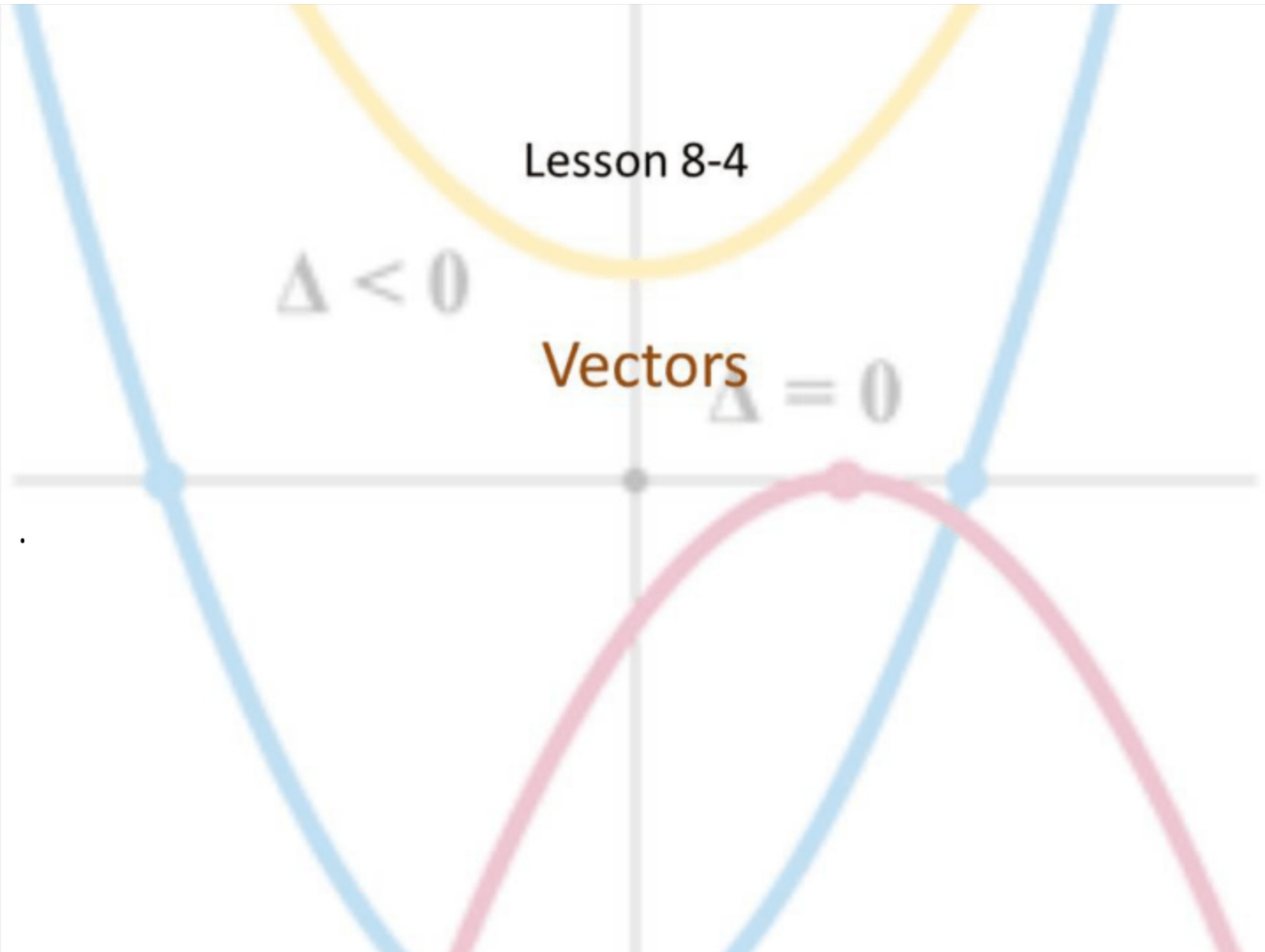
$\frac{r^4}{r^2} = \frac{2r^2\cos\theta\sin\theta}{r^2}$

$\sqrt{r^2} = \sqrt{2\cos\theta\sin\theta}$

$\boxed{r = \pm\sqrt{2\cos\theta\sin\theta}}$

# Lesson 8-4

## Vectors



# Objective

Students will…

- Be able to define vectors and differentiate ~~it~~ them. from a scalar.
- Be able to represent vectors in component form.
- Be able to add and subtract vectors, and multiply vectors by a scalar.

# Scalar and Vectors

In mathematics, certain quantities are determined completely by their **magnitude**, such as length, mass, area, and temperature. For example, a length of 5 meters can be adequately represented by a single number. Such quantity is called a **scalar**.

However, some quantities need to be accompanied by the **direction** of its magnitude. For example, when studying a rocket blasting off, the speed or the velocity alone is not enough. We would need to make sure that the rocket is also travelling in the right direction. These types of quantities are represented through **vectors**.

# Vectors

**Vector**- A line segment in a plane with an assigned direction.

Ex.

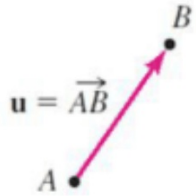
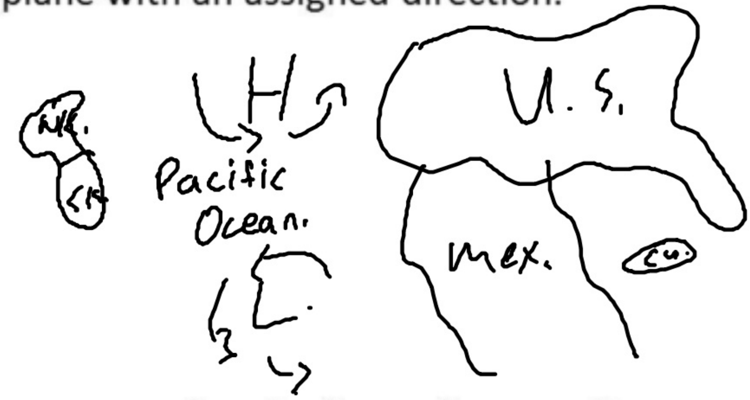



**Figure 1**




As shown in figure 1 above, a vector takes the form of a **ray**, a line segment with a point at one end and an arrow on the other. The arrow indicates the direction of the movement.

Also, every vector has a starting point, or an **initial point**, as well as an ending point, or a **terminal point**. In the picture above, point A is the initial point, while point B is the terminal point.

# Vectors

A **magnitude** of a vector is the length of the vector.
So, in figure 1, the magnitude of the vector $\boldsymbol{u}$, denoted, $\boxed{|\boldsymbol{u}|,}$ is the length AB.

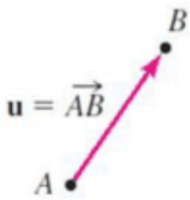


**Figure 1**

Now, as we can see, vectors always contain two points, **initial** and **terminal**. Therefore, vectors are most significantly studied on coordinate planes (the "x, y plane").

# Vectors in the Coordinate Plane (x, y).

Using the coordinate plane, vectors are often represented as ordered pairs, but with < and >, instead of parenthesis. So, for example, we can represent the vector **v** as,

> $v = \langle a, b \rangle$, where $a$ is the **horizontal component**, i.e. the amount of movement along the $x$-axis, and $b$ is the **vertical component**, i.e. the amount of movement along the $y$-axis.

That being said, if a vector **v** is represented in the plane with initial point $P(x_1, y_1)$ and terminal point $Q(x_2, y_2)$, then

$$\boxed{v = \langle x_2 - x_1 \,,\; y_2 - y_1 \rangle}$$

This is known as the **<u>component form</u>**.

# Example

Find the component form of the vector **u** with initial point $(2, -5)$, and terminal point $(3, 7)$.

comp. form $= \langle x_2 - x_1, y_2 - y_1 \rangle$

$u = \langle 3-2, 7 - -5 \rangle = \boxed{\langle 1, 12 \rangle}$

If the vector $\mathbf{v} = \langle 3, 7 \rangle$ is sketched with initial point $(2, 4)$, what is its terminal point?

$\langle 3, 7 \rangle = \langle x_2 - x_1, y_2 - y_1 \rangle = \langle x_2 - 2, y_2 - 4 \rangle$

Terminal $= (5, 11)$

$x_2 - 2 = 3$

$x_2 = 5$

$y_2 - 4 = 7$

$y_2 = 11$

# Magnitude of the Vector

The magnitude or length of a vector $\mathbf{v} = \langle a, b \rangle$ is

$$|\mathbf{v}| = \sqrt{a^2 + b^2}$$

Ex. Find the magnitude of each vector.

a. $\mathbf{u} = \underset{}{\overset{\langle a,\ b \rangle}{\langle 2, -3 \rangle}}$

$|u| = \sqrt{a^2+b^2}$

$= \sqrt{2^2+(-3)^2}$

$= \sqrt{4+9}$

$= \boxed{\sqrt{13}}$

b. $\mathbf{v} = \langle 5, 0 \rangle$

$|v| = \sqrt{a^2+b^2}$

$= \sqrt{5^2+0^2}$

$= \sqrt{25}$

$= \boxed{5}$

c. $\mathbf{w} = \left\langle \frac{3}{5}, \frac{4}{5} \right\rangle$

$|w| = \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2}$

$= \sqrt{\frac{9}{25} + \frac{16}{25}}$

$= \sqrt{\frac{25}{25}}$

$= \sqrt{1} = 1$

$x(a+b)$
$xa+xb.$

# Vector Operations

Another usefulness of writing vectors in component form is the ease of doing basic operations. Here is how to perform the three basic mathematical operations using vectors:

If $\mathbf{u} = \langle a_1, b_1 \rangle$ and $\mathbf{v} = \langle a_2, b_2 \rangle$, then

$\mathbf{u} + \mathbf{v} = \langle a_1 + a_2, b_1 + b_2 \rangle$ **(Addition)**

$\mathbf{u} - \mathbf{v} = \langle a_1 - a_2, b_1 - b_2 \rangle$ **(Subtraction)**

$c\mathbf{u} = \langle ca_1, cb_1 \rangle$, where $c \in \mathbb{R}$ **(Scalar Multiplication)**

We will be dealing with vector multiplication (i.e. **uv**) in the next section.

# Example

If $\mathbf{u} = \langle 2, -3 \rangle$ and $\mathbf{v} = \langle -1, 2 \rangle$, find $\mathbf{u} + \mathbf{v}$, $\mathbf{u} - \mathbf{v}$, $2\mathbf{u} + 3\mathbf{v}$, $2\mathbf{u} - 3\mathbf{v}$

$u + v = \langle 2 + {}^{-}1, {}^{-}3 + 2 \rangle$

$= \boxed{\langle 1, -1 \rangle}$

$u - v = \langle 2 - {}^{-}1, -3 - 2 \rangle$

$= \boxed{\langle 3, -5 \rangle}$

$2u + 3v = 2\langle 2, -3 \rangle + 3\langle -1, 2 \rangle$

$= \langle 4, -6 \rangle + \langle -3, 6 \rangle$

$= \boxed{\langle 1, 0 \rangle}$

$2u - 3v = \langle 4, -6 \rangle - \langle -3, 6 \rangle$

$= \boxed{\langle 7, {}^{-}12 \rangle}$

$0 = \langle 0, 0 \rangle$

# Properties of Vectors

The properties of vectors aren't that different from properties of real numbers.

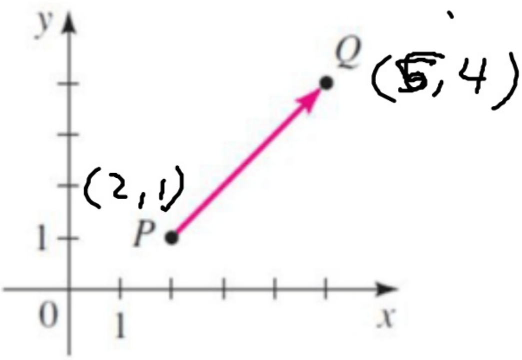
**Properties of Vectors**

| Vector addition | Multiplication by a scalar |
|---|---|
| $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$ | $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$ |
| $\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$ | $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$ |
| $\mathbf{u} + \mathbf{0} = \mathbf{u}$ | $(cd)\mathbf{u} = c(d\mathbf{u}) = d(c\mathbf{u})$ |
| $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$ | $1\mathbf{u} = \mathbf{u}$ |
| Length of a vector | $0\mathbf{u} = \mathbf{0}$ |
| $\lvert c\mathbf{u} \rvert = \lvert c \rvert \lvert \mathbf{u} \rvert$ | $c\mathbf{0} = \mathbf{0}$ |

# Homework Problem

Express the vector with initial point $P$ and terminal point $Q$ in component form. $= \langle x_2 - x_1 , y_2 - y_1 \rangle$

**7.**




$\langle 5-2 , 4-1 \rangle$

$= \boxed{\langle 3 , 3 \rangle}$

# Homework Problem

Find 2**u**, $-3$**v**, **u** + **v**, and 3**u** $-4$**v** for the given vectors **u** and **v**.

**19.** $\mathbf{u} = \langle 0, -1 \rangle, \quad \mathbf{v} = \langle -2, 0 \rangle$

$2u = 2\langle 0, -1 \rangle$

$= \boxed{\langle 0, -2 \rangle}$

$-3v = 3\langle -2, 0 \rangle$

$= \boxed{\langle 6, 0 \rangle}$

$u + v = \langle -2, -1 \rangle.$

$3u - 4v = \langle +8, -3 \rangle.$

# Homework Problem

Find $|\mathbf{u}|$, $|\mathbf{v}|$, $|2\mathbf{u}|$, $\left|\frac{1}{2}\mathbf{v}\right|$, $|\mathbf{u}+\mathbf{v}|$, $|\mathbf{u}-\mathbf{v}|$, and $|\mathbf{u}|-|\mathbf{v}|$

**26.** $\mathbf{u} = \langle -6, 6 \rangle$, $\mathbf{v} = \langle -2, -1 \rangle$

$|u| = \sqrt{(-6)^2 + 6^2}$

$= \boxed{\sqrt{72}}$

$= \sqrt{9} \cdot \sqrt{4} \cdot \sqrt{2}$

$= 6\sqrt{2}$

$|v| = \sqrt{(-2)^2 + (-1)^2}$

$= \sqrt{4 + 1}$

$= \boxed{\sqrt{5}}$

$\left|\frac{1}{2}v\right| = \left|\frac{1}{2}\right| |v| = \boxed{\frac{\sqrt{5}}{2}}$

$|2u| = |2|\,|u| = 2 \cdot 6\sqrt{2} = \boxed{12\sqrt{2}}$

$|u| - |v| = \boxed{6\sqrt{2} - \sqrt{5}}$

# Homework 4/10

TB pg. 615 #7-15 (odd), 17, 19, 25, 26