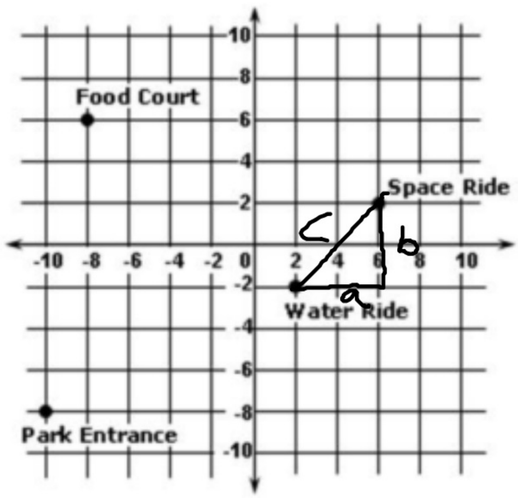
Warm Up 4/8



Find the coordinates for each location.

Food Court - (-8, 6)

Space Ride - (6, 2)

Water Ride - (2, -2)

Entrance - (-10, -8)

Can you tell the distance between any pair of locations from these coordinates right away? NO

How would you find the distance between any pair (i.e. from the water ride to the space ride)?

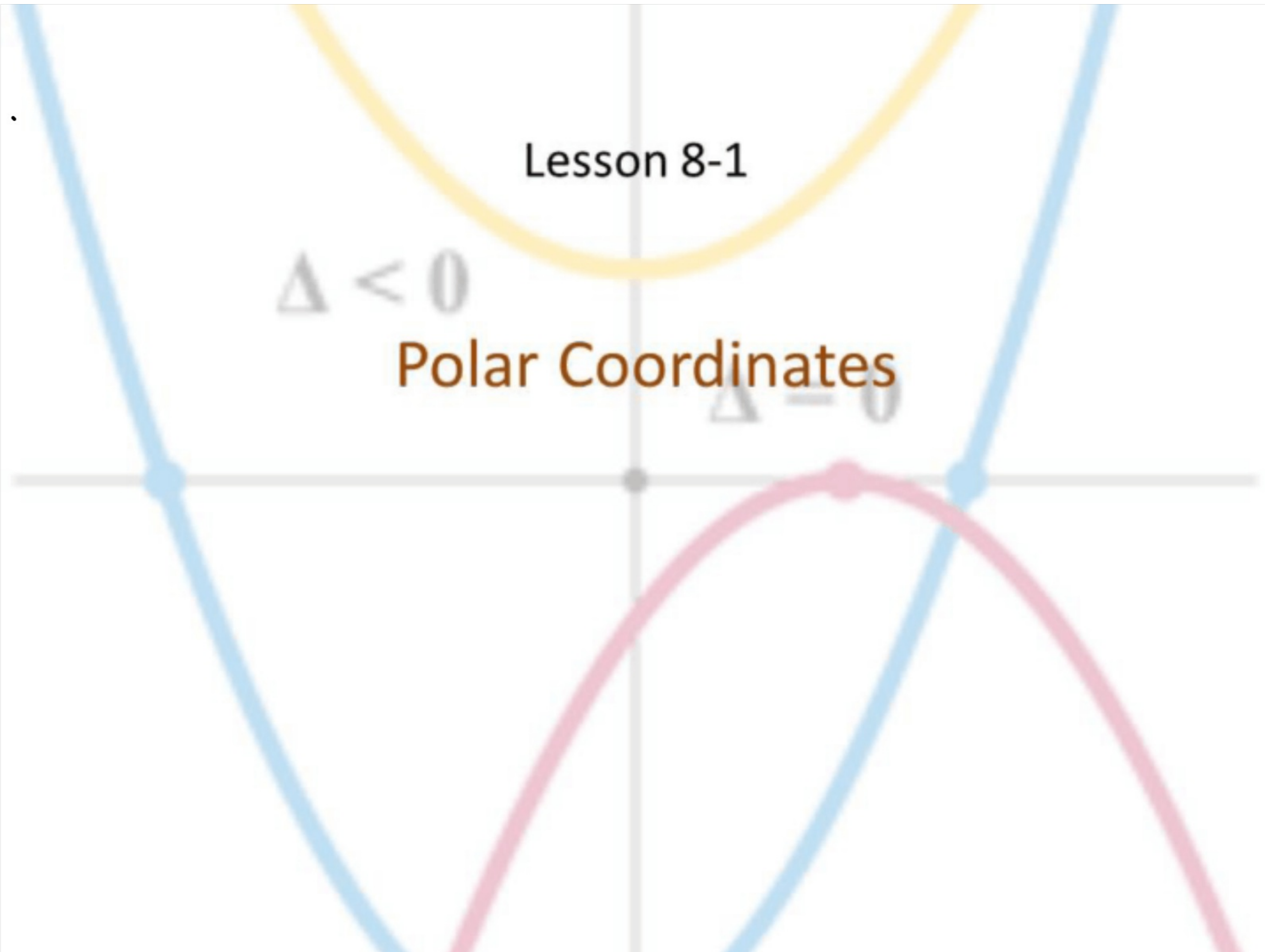
Dist Formula = $\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$

$a^2+b^2=c^2$

$c=\sqrt{a^2+b^2}$

Lesson 8-1

Polar Coordinates



Objective

Students will…

- Be able to define polar coordinates.
- Be able to find the relationship between polar and rectangular coordinates.
- Be able to convert equations into different coordinate systems.

Rectangular vs Polar Coordinates

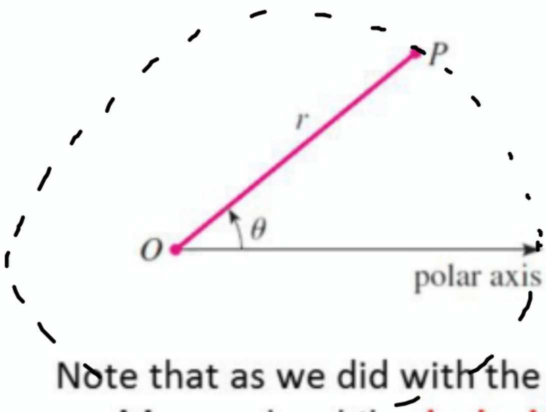
We should all be familiar with *rectangular coordinates*, otherwise known as Cartesian coordinates, that are represented in **ordered pairs**, (x, y). As we studied back in geometry and algebra, rectangular coordinates have their uses in mathematics.

However, as we have just discovered, rectangular coordinates do have limitations. They may be useful in giving us a rather accurate and easy-to-follow location of points, they do not give much information on their locations **relative to other points**. For this, **polar coordinates** are much better.

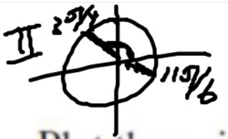
Polar Coordinates

Polar Coordinates- coordinates that use distances and directions to specify the location of a point on a plane. They are given in ordered pair form (r, θ), where r is the distance between the two points, and θ is the angle between polar axis and the segment connecting the two points.

ex.



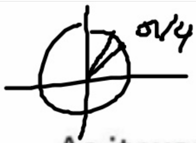
Note that as we did with the *unit circle*, **counter-clockwise** direction is **positive** angle while **clockwise** is **negative**. Also, θ is measured in **radians**.



Plotting Polar Coordinates

Plot the points whose polar coordinates are given.

(a) $(1, 3\pi/4)$ (b) $(3, -\pi/6)$ (c) $(3, 3\pi)$ (d) $(-4, \pi/4)$



Different Coordinates for the Same Point

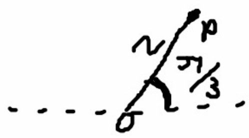
As it was for the points on the *unit circle*, since polar coordinates are viewed in **circular motion**, there are **infinitely many** polar coordinates for each point. This can simply be done by adding or subtracting the angle, θ, by 2π.

$\frac{\pi}{3} + \frac{6\pi}{3}$

Ex. (a) Graph the point with polar coordinates $P(2, \pi/3)$.

(b) Find two other polar coordinate representations of P

a).



$P = (2, \pi/3)$

$= (2, 7\pi/3)$

$= (2, -\frac{5\pi}{3})$

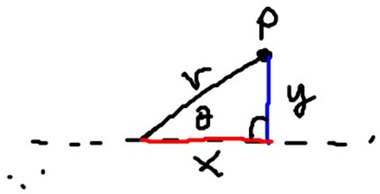
$\frac{\pi}{3} - \frac{6\pi}{3}$

$= \frac{-5\pi}{3}$.

Converting Polar into Rectangular Coordinates

$\pi/4$ $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

Consider the graph of the general polar coordinate, (r, θ). Soh Cah Toa.



$(r)\cos\theta = \frac{x}{r}(r)$ $\quad$ $(r)\sin\theta = \frac{y}{r}(r)$

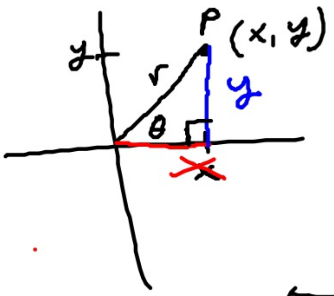
$$x = r\cos\theta \qquad y = r\sin\theta$$

$$(x, y) = (r\cos\theta, r\sin\theta)$$

Converting Rectangular into Polar Coordinates

Soh Cah Toa.

Consider the graph of the general rectangular coordinate, (x, y).



$$\sqrt{x^2+y^2}=\sqrt{r^2}$$

$$r=\sqrt{x^2+y^2}$$

$$\tan^{-1}(\tan\theta)=\tan^{-1}\left(\frac{y}{x}\right)$$

$$\theta=\tan^{-1}\left(\frac{y}{x}\right)$$

$$\text{Polar}=(r,\theta)$$

$$=\left(\sqrt{x^2+y^2},\ \tan^{-1}\left(\frac{y}{x}\right)\right).$$

Recap

So, for recap…

Relationship between Polar and Rectangular Coordinates

1. To change from polar to rectangular coordinates, use the formulas

$$x = r\cos\theta \qquad \text{and} \qquad y = r\sin\theta$$

2. To change from rectangular to polar coordinates, use the formulas

$$r^2 = x^2 + y^2 \qquad \text{and} \qquad \tan\theta = \frac{y}{x} \quad (x \neq 0)$$

Examples

$\frac{2\pi}{3} = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

Find the rectangular coordinates for the point that has polar coordinates $\left(4, \frac{2\pi}{3}\right)$

Polar → r, θ

(x, y)?

$x = r\cos\theta \quad y = r\sin\theta$

$(x, y) = \left(4\cos\frac{2\pi}{3}, 4\sin\frac{2\pi}{3}\right)$

$= \left(4\left(-\frac{1}{2}\right), 4\left(\frac{\sqrt{3}}{2}\right)\right) = \boxed{\left(-2, 2\sqrt{3}\right)}$

Find the rectangular coordinates for the point that has polar coordinates $\left(-4, \frac{5\pi}{2}\right)$

Polar → r, θ

$(r\cos\theta, r\sin\theta) = \left(-4(0), -4(1)\right) = \boxed{(0, -4)}$

$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ $\tan^{-1}(-1)$

Example

Find the polar coordinates for the point that has rectangular coordinates $(2, -2)$

rect → x, y

(r, θ).

$$(r, \theta) = \left(\sqrt{x^2+y^2}, \tan^{-1}\left(\frac{y}{x}\right)\right)$$

$$= \left(\sqrt{2^2+(-2)^2}, \tan^{-1}\left(\frac{-2}{2}\right)\right) = \left(\sqrt{8}, \frac{3\pi}{4}\right) \text{ or } \left(\sqrt{8}, \frac{3\pi}{4}\right).$$

Find the polar coordinates for the point that has rectangular coordinates $(-7, -7)$

Homework Problems

Plot the point that has the given polar coordinates.

1. $\left(4, \frac{\pi}{4}\right)$

3. $\left(6, -\frac{7\pi}{6}\right)$

Homework Problems

Give two other polar coordinate representations of the following point.

7. $\left(3, \frac{\pi}{2}\right)$

12. $(3, 1)$

Homework 4/7

TB pg. 586-587 ~~#1, 3, 7, 12~~, 25-39 (odd)