Warm Up 3/10

Evaluate the following:

1. $\sin(45°)$

2. $\cos\frac{\pi}{3}$

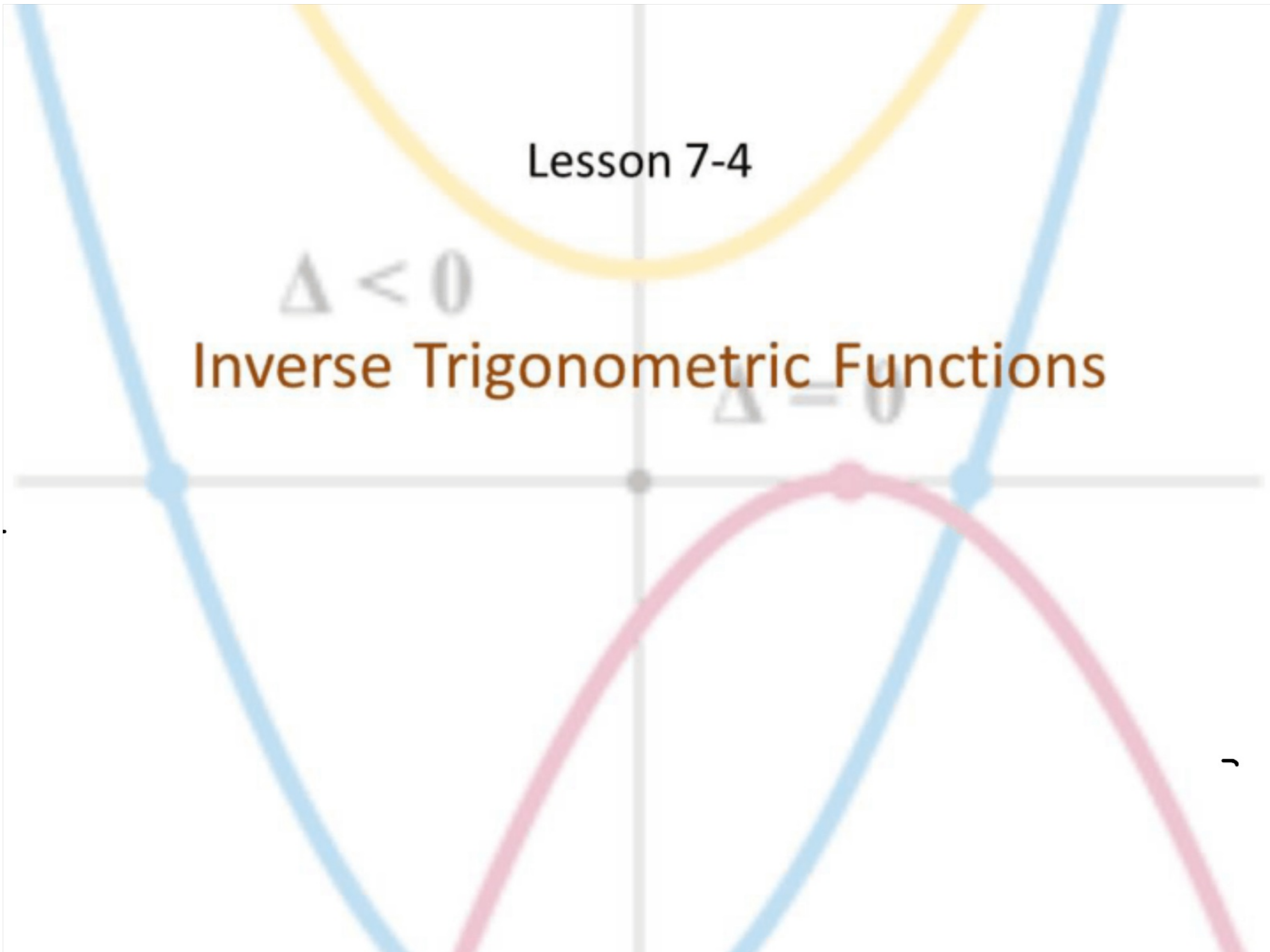
3. $\tan\frac{3\pi}{4}$

$\frac{\sin}{\cos} = \frac{y}{x}\ \frac{+}{-}$

$= \boxed{\frac{\sqrt{2}}{2}}$

$= \boxed{\frac{1}{2}}$

$= \boxed{-1}$


Lesson 7-4
Δ < 0
Inverse Trigonometric Functions
Δ = 0

Objective

Students will…

- Be able to explain what the inverse sine, cosine, and tangent functions do.
- Be able to evaluate angle measurement using the inverse trig functions.

Inverse Functions

We have dealt with inverse functions before when learning about the polynomial functions. As it was for the polynomial functions, there exists an inverse function for the trig functions as well. The core meaning behind these functions haven't changed: Inverse functions, "undo" their original function. In other words…

For an **one-to-one** function f, if $f(x) = y$, then $f^{-1}(y) = x$.

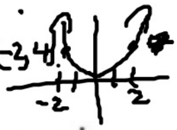
f^{-1} f^{-1}

Here, we see that to "undo" the function f (or solve for x), we simply take the inverse function f^{-1} to "both" sides.

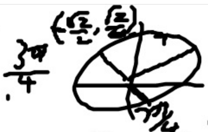
$f(x) = x^2$

$f(2) = 2^2 = 4$

$f(-2) = (-2)^2 = 4$



One-to-One



$\frac{3\pi}{4}\ \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$\pi/4\ \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$\sin(\pi/4) = \frac{\sqrt{2}}{2}$

$\sin(3\pi/4) = \sqrt{2}/2$

Moreover, we have also learned that only **one-to-one** functions can have an inverse function, which means that for every output, there is exactly only **one** input. This is CLEARLY not the case for trig functions. Just think back to the unit circle! Values like $\frac{\sqrt{2}}{2}$ happen twice for both sine and cosine functions. Thus, trig functions do **not** have an inverse function as a whole. HOWEVER, we can simply restrict the functions to a certain quadrant, and use the inverse relationship.

Note: This is exactly why we get an answer when we use the inverse trig function on our calculators. They are programmed to restrict the functions to only **two** quadrants.

Example

Find the exact value of the expression. For sine, limit the function to Quadrants [redacted] I, IV and for cosine, limit the function to Quadrants I, II

1. $\sin^{-1}\frac{\sqrt{3}}{2}$

$= 60°, \boxed{\frac{\pi}{3}}$

2. $\cos^{-1}\frac{\sqrt{3}}{2}$

$30°, \boxed{\frac{\pi}{6}}$

3. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$150°, \boxed{\frac{5\pi}{6}}$

Homework Problems

Use a calculator to find an approximate value of each expression correct to **five** decimal places, if it is defined.

9a. $\sin^{-1}(0.13844)$

$= \boxed{0.13889}$

9b. $\cos^{-1}(-0.92761)$

$= \boxed{2.75876}$

12a. $\cos^{-1}(-0.25713)$

$= \boxed{1.83085}$

12b. $\tan^{-1}(-0.25713)$

$= \boxed{-0.25168}$

Example

Find the exact value of the expression if it is defined.

$\cancel{\cos\left(\cos^{-1}\right.}\left.\frac{2}{3}\right)$

$= \boxed{2/3}$

$\cancel{\tan^{-1}\left(\tan\right.}\left.\frac{\pi}{6}\right)$

$= \boxed{\frac{\pi}{6}}$

Example

Find the exact value of the expression if it is defined.

$\tan\left(\sin^{-1}\frac{\sqrt{2}}{2}\right)$

$\tan\left(\frac{\pi}{4}\right) = \frac{y}{x}$

$= \boxed{1}$

$\cos^{-1}\left(\sqrt{3}\sin\frac{\pi}{6}\right)$

$= \cos^{-1}\left(\sqrt{3}\cdot\frac{1}{2}\right)$

$= \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$= \boxed{\frac{\pi}{6}}$

Homework Problems

Find the exact value of the expression if it is defined.

19. $\cancel{\sin^{-1}}\left(\cancel{\sin}\left(-\frac{\pi}{6}\right)\right)$

$\boxed{-\frac{\pi}{6}}$

25. $\cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)$

$\cos\left(\frac{\pi}{3}\right) = \boxed{\frac{1}{2}}$

Homework 3/10

TB pg. 557 #1, 3, 6, 9-12, 13, 15, 19, 23, 25, 26, 27