Warm Up 3/7

Write the double angle and half-angle formulas for sine, cosine, and tangent:

$\sin(2x) = 2\sin x\cos x$

$\cos(2x) = \cos^2 x - \sin^2 x$

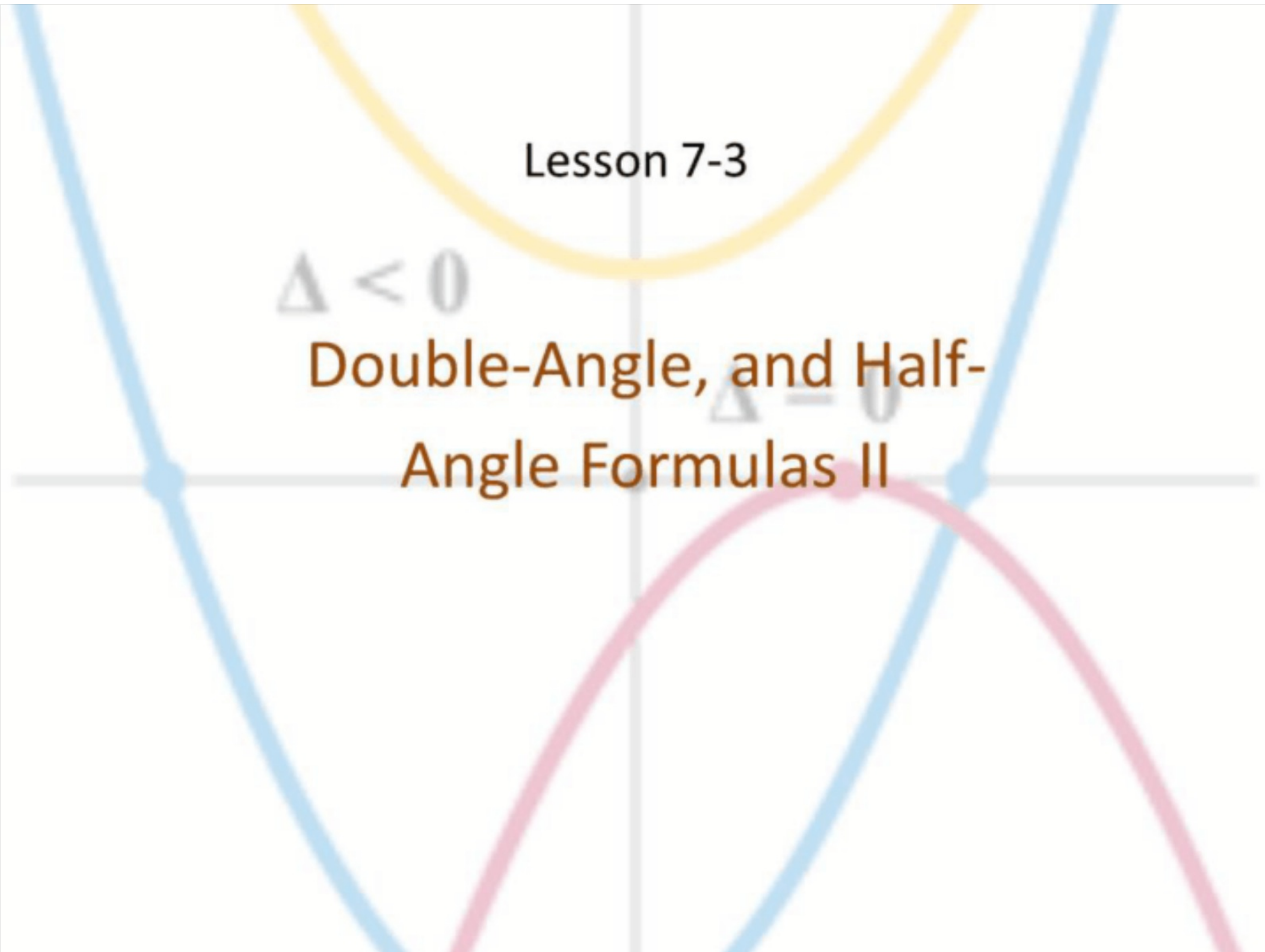
$= 1 - 2\sin^2 x$

$= 2\cos^2 x - 1$

$\tan(2x) = \dfrac{2\tan x}{1-\tan^2 x}$

Lesson 7-3

Double-Angle, and Half-Angle Formulas II



Objective

Students will…

- Be able to apply the Double-Angle and Half-Angle formulas to verify identities.

Guidelines for Proving Identities

1. Always look for opportunities to use the Sum-to-Product formulas, before applying the Double-Angle formulas.

2. When either can be used, it's **usually** best to use the Double-Angle formulas, rather than the Half-Angle formulas. Half-Angle formulas are rarely used when proving identities.

3. With regards to the Double-Angle formulas, always look for multiples of 2. When whole numbers are doubled, they are always even (i.e. multiples of 2!).

$3x = 2x + x$

Example

$yx - x = x(y-1)$

$\frac{x}{3} + \frac{y}{3} = \frac{x+y}{3}$.

Prove the identity: $\frac{\sin 3x}{\sin x \cos x} = 4\cos x - \sec x$

$\frac{x+y}{3} = \frac{x}{3} + \frac{y}{3}$.

$$LHS = \frac{\sin(2x+x)}{\sin x \cos x} = \frac{\sin 2x \cos x + \cos 2x \sin x}{\sin x \cos x}$$

$$= \frac{(2\sin x \cos x)\cos x + (2\cos^2 x - 1)\sin x}{\sin x \cos x}.$$

$$= \frac{2\sin x \cos^2 x + 2\cos^2 x \sin x - \sin x}{\sin x \cos x} = \frac{4\cos^2 x \sin x - \sin x}{\sin x \cos x}.$$

$$= \frac{\cancel{\sin x}(4\cos^2 x - 1)}{\cancel{\sin x}\cos x} = \frac{4\cos^2 x - 1}{\cos x} = \frac{4\cos^{\cancel{2}} x}{\cancel{\cos x}} - \frac{1}{\cos x}$$

$$= 4\cos x - \sec x = RHS.$$

$\sin(2(x)) = 2\sin x \cos x$

Homework Problems

Prove the identity.

60. $\sin 8x = 2 \sin 4x \cos 4x$

$LHS = \sin(2(4x)) = 2\sin 4x \cos 4x = RHS$

Homework Problems

Prove the identity.

62. $\frac{2\tan x}{1+\tan^2 x} = \sin 2x$

LHS $= \dfrac{2\left(\frac{\sin x}{\cos x}\right)}{\frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}} = \dfrac{\frac{2\sin x}{\cos x}}{\frac{\cos^2 x + \sin^2 x}{\cos^2 x}}$

$= \dfrac{\frac{2\sin x}{\cos x}}{\frac{1}{\cos^2 x}} = \dfrac{2\sin x}{\cancel{\cos x}} \cdot \dfrac{\cos^2 x}{1}$

$= 2\sin x\cos x$

RHS: $\sin(2x) = 2\sin x\cos x.$

✓

Homework Problems

Prove the identity.

64. $\frac{1+\sin 2x}{\sin 2x} = 1 + \frac{1}{2}\sec x \csc x$

$$LHS = \frac{1+2\sin x\cos x}{2\sin x\cos x}$$

$$RHS = 1 + \frac{1}{2}\left(\frac{1}{\cos x}\right)\left(\frac{1}{\sin x}\right)$$

$$= \frac{1}{1} + \frac{1}{2\cos x\sin x}$$

$$\frac{2\cos x\sin x}{2\cos x\sin x} + \frac{1}{2\cos x\sin x}.$$

$$= \frac{2\cos x\sin x + 1}{2\cos x\sin x}.$$

✓

Homework Problems

66. $\cot 2x = \frac{1-\tan^2 x}{2\tan x}$

$$LHS = \frac{1}{\tan 2x}$$

$$= \frac{1}{\frac{2\tan x}{1-\tan^2 x}}$$

$$= \frac{1-\tan^2 x}{2\tan x} = RHS.$$

Homework 3/7

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