PreCalculus Name: Period: Date:

Warm Up 3/5

Lesson 7-3: Double-Angle, and Half-Angle Formulas

Objectives
Students will…
- Be able to know and derive the Double-Angle and Half-Angle formulas of sine, cosine, and tangent.
- Be able to apply the Double-Angle and Half-Angle formulas.

Double-Angle Formulas
The following formulas are direct results of addition and subtraction formulas. **Double-Angle** formulas allows us to find the values of the trigonometric functions at $2x$ from their values at x.

Double-Angle Formulas:

For Sine: $\sin(2x) = 2\sin x \cos x$

For Cosine: $\cos(2x) = \cos^2 x - \sin^2 x$
$= 1 - 2\sin^2 x$
$= 2\cos^2 x - 1$

For Tangent: $\tan(2x) = \frac{2\tan x}{1-\tan x}$

Proof of Double-Angle Formulas

Prove the formula: $\cos(2x) = \cos^2 x - \sin^2 x$

Show that $\cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

Prove the formula: $\sin(2x) = 2\sin x \cos x$

Prove the formula: $\tan(2x) = \frac{2\tan x}{1-\tan^2 x}$

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Using the Double-Angle Formulas

If $\cos x = -\frac{2}{3}$ and x is in quadrant II, find $\cos 2x$, $\sin 2x$, and $\tan 2x$.

Half-Angle Formulas

The next set of formulas relate the values of trig functions at $\frac{1}{2}x$ to their values at x.

They are known as the **Half-Angle Formulas**.

Half-Angle Formulas:

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}} \qquad \cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}} \qquad \tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{\sin u}{1+\cos u}$$

*The choice of $+$ or $-$ depends on which quadrant $\frac{u}{2}$ lies in.

Using Half-Angle Formulas

Find the exact value of $\sin 22.5°$

Find $\tan\frac{u}{2}$ if $\sin u = \frac{2}{5}$ and u is in quadrant II.

Homework 3/5
TB pg. 548 #1-7 (odd), 15, 17, 21, 23, 35, 36, 37