$(x+y)(x-y)=x^2-y^2$

Warm Up 3/4

Verify the identity.

$$\cos(x+y)\cos(x-y)=\cos^2 x-\sin^2 y$$

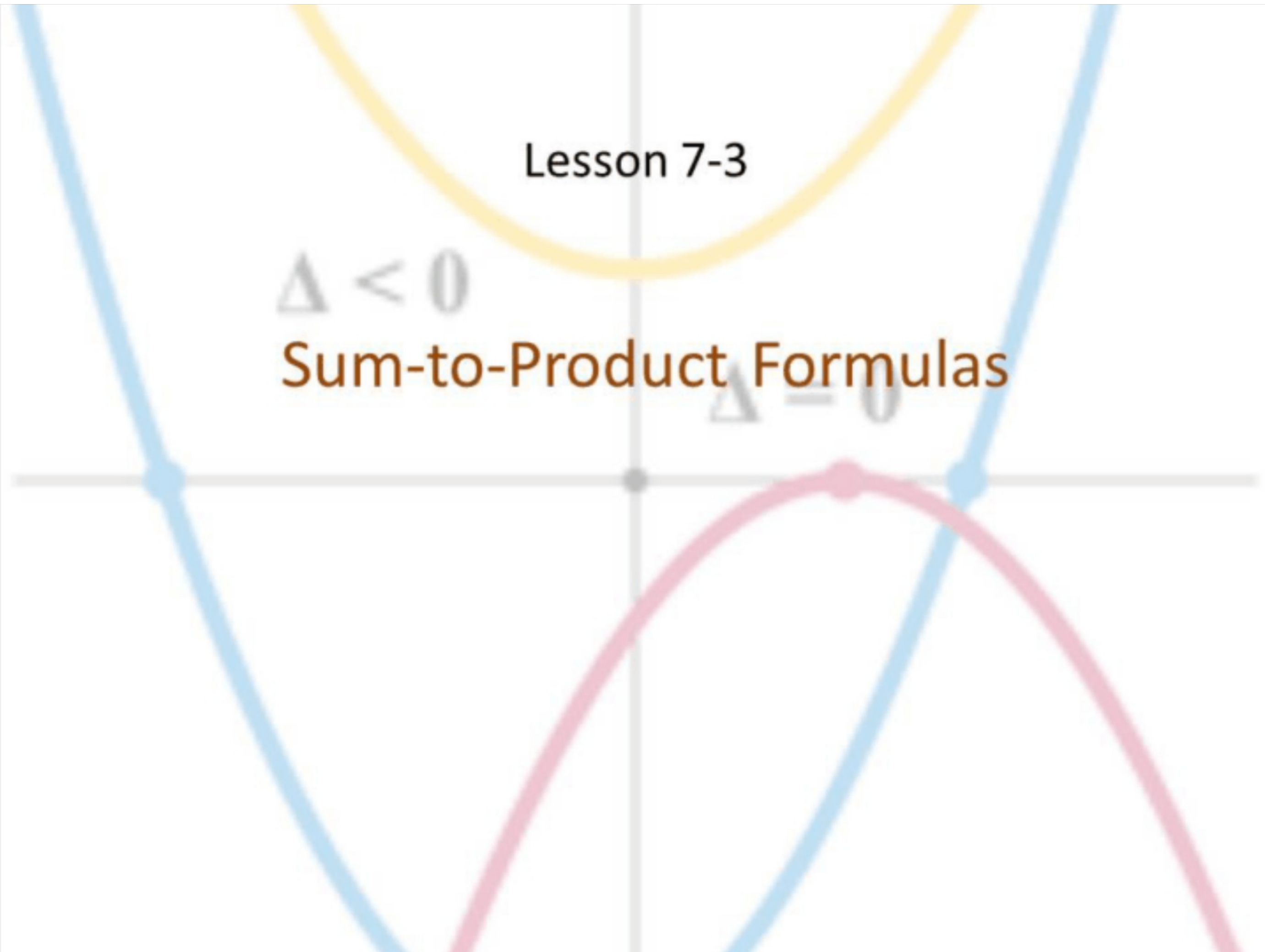
$LHS=(\cos x\cos y-\sin x\sin y)(\cos x\cos y+\sin x\sin y)$

$=\cos^2 x\cos^2 y-\sin^2 x\sin^2 y$

$=\cos^2 x(1-\sin^2 y)-\sin^2 y(1-\cos^2 x)$

$=\cos^2 x\cancel{-\sin^2 y\cos^2 x}-\sin^2 y\cancel{+\sin^2 y\cos^2 x}$

$=\cos^2 x-\sin^2 y=RHS$ ✓


Lesson 7-3
Δ < 0
Sum-to-Product Formulas

Objective

Students will…

- Be able to know the Sum-to-Product Formulas.
- Be able to use the Sum-to-Product formulas to prove identities.

+ −

Sum-to-Product Formulas

We now move further into different formulas for trig functions. The following is the **Sum-to-Product** formulas, which do exactly as ~~it~~ they say~~s~~ turn sums (addition or subtraction) into products (multiplication).

Sum-to-Product Formulas:

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$$

$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$$

Using Sum-to-Product Formulas

Write $\sin 7x + \sin 3x$ as a product.

$= 2\sin\frac{7x+3x}{2}\cos\frac{7x-3x}{2} = 2\sin\frac{10x}{2}\cos\frac{4x}{2}$

$= \boxed{2\sin 5x\cos 2x}$

Write $\sin 11x + \sin 5x$ as a product.

$= \boxed{2\sin 8x\cos 3x}$

$2\sin\frac{11x+5x}{2}\cos\frac{11x-5x}{2} = 2\sin\frac{16x}{2}\cos\frac{6x}{2} =$

Homework Problem

Write the sum as a product.

49. $\cos 4x - \cos 6x$

$$= -2\sin\frac{4x+6x}{2}\sin\frac{4x-6x}{2} = -2\sin(5x)\sin(-x)$$

$$= \boxed{2\sin 5x \sin x}$$

$$\frac{f(x)}{g(x)}$$

Using Sum-to-Product Formulas

Verify the identity: $\frac{\sin 3x - \sin x}{\cos 3x + \cos x} = \tan x$

$$LHS = \frac{\cancel{2}\cancel{\cos\frac{3x+x}{2}}\sin\frac{3x-x}{2}}{\cancel{2}\cancel{\cos\frac{3x+x}{2}}\cos\frac{3x-x}{2}}$$

$$= \frac{\sin\left(\frac{2x}{2}\right)}{\cos\left(\frac{2x}{2}\right)} = \frac{\sin x}{\cos x} = \tan x = RHS \checkmark$$

Using Sum-to-Product Formulas

Verify the identity: $\frac{\sin 4x+\sin 2x}{\sin 2x} = \frac{\sin 3x}{\sin x}$

Homework Problems

Verify the identity: 71. $\frac{\sin x+\sin 5x}{\cos x+\cos 5x} = \tan 3x$

Homework Problems

$\sin(x+y)$

79. Show that $\sin 45° + \sin 15° = \sin 75°$

$$= 2\sin\frac{45°+15°}{2}\cos\frac{45°-15°}{2}$$

$$= 2\sin 30°\cos 15°$$

$$= 2\sin 30°\cos(45-30)$$

$$= 2\sin 30°(\cos 45°\cos 30° + \sin 45°\sin 30°)$$

$$= 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2}\right)$$

$$= \left(\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2}\right)$$

$$\sin 75 = \sin(45+30)$$

$$\sin 45°\cos 30° + \cos 45°\sin 30°$$

$$= \left(\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2}\right)$$

✓

Homework 3/4

TB pg. 548-549 #47-53 (odd), 72, 74, 75, 77

$\cos(-x) = \cos x.$

72. $$\frac{\sin 3x + \sin 7x}{\cos 3x - \cos 7x} = \cot 2x$$

$$LHS = \frac{2\sin\frac{3x+7x}{2}\cos\frac{3x-7x}{2}}{-2\sin\frac{3x+7x}{2}\sin\frac{3x-7x}{2}} = \frac{\cancel{2}\,\cancel{\sin(5x)}\cos(-2x)}{-\cancel{2}\,\cancel{\sin(5x)}\sin(-2x)}.$$

$$= \frac{\cos(\cancel{-}2x)}{-\sin(-2x)} = \frac{\cos 2x}{\sin 2x} = \cot 2x = RHS$$