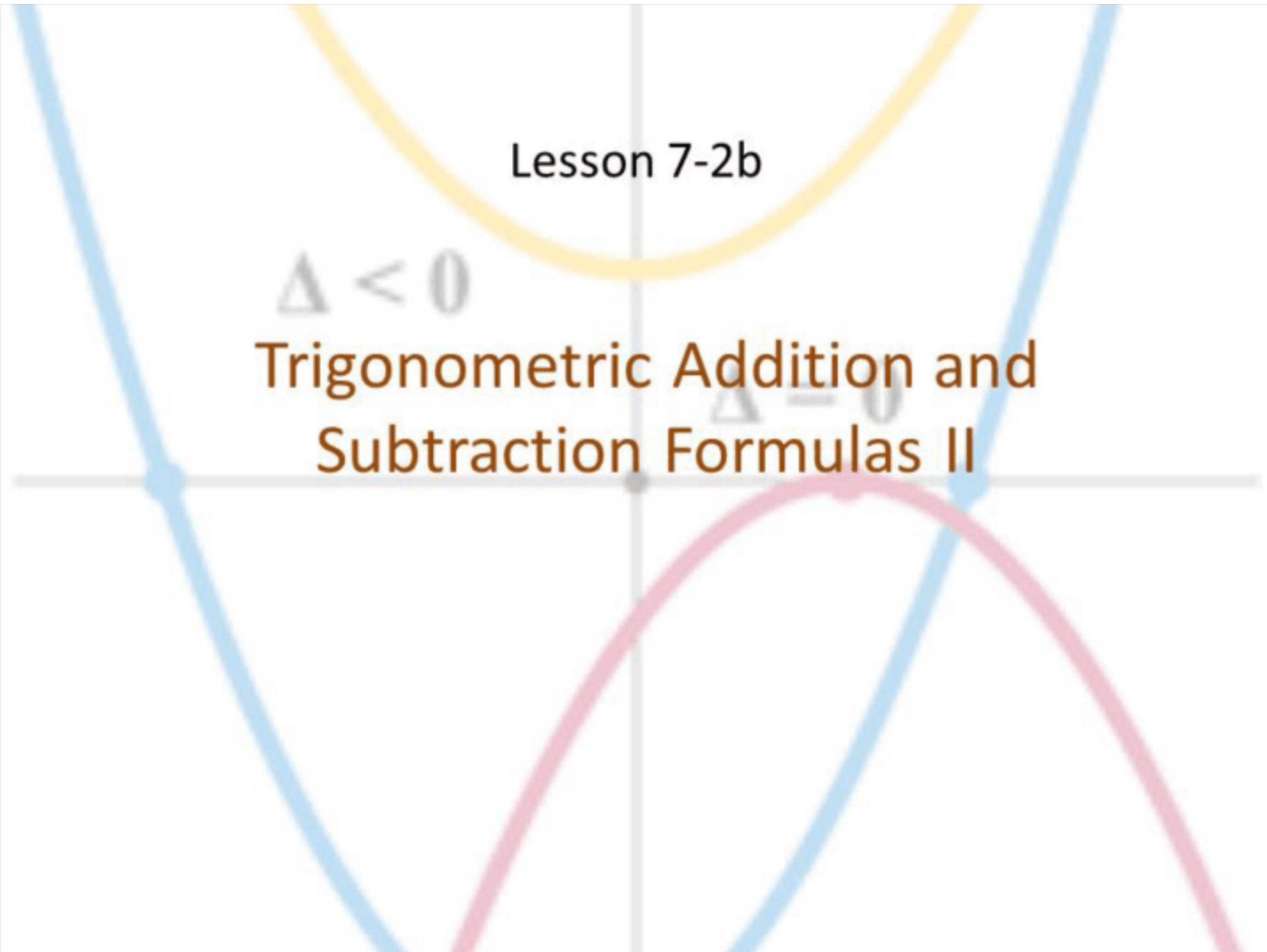

Lesson 7-2b
Δ < 0
Trigonometric Addition and
Subtraction Formulas II

Objective

Students will…

- Be able to use addition and subtraction formulas to evaluate trig functions and to prove or verify identities.

Addition and Subtraction Formulas

Formulas for Sine:

$$\sin(s + t) = \sin s \cos t + \cos s \sin t$$

$$\sin(s - t) = \sin s \cos t - \cos s \sin t$$

Formulas for Cosine:

$$\cos(s + t) = \cos s \cos t - \sin s \sin t$$

$$\cos(s - t) = \cos s \cos t + \sin s \sin t$$

Formulas for Tangent:

$$\tan(s + t) = \frac{\tan s + \tan t}{1 - \tan s \tan t}$$

$$\tan(s - t) = \frac{\tan s - \tan t}{1 + \tan s \tan t}$$

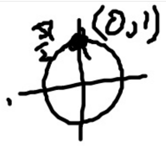
Guidelines for Proving Identities

Furthermore, we have some guidelines/tips for proving identities.

1. **Focus on the fractions**: More often than not, identity proofs are more easily done when you work with the side that involves a fraction.

2. **Pick the more "complicated" side**: It's easier to modify the sides that has less sines or cosines. Generally, rewriting everything as sine or cosine can help you when you are "stuck."

3. **Use the Known Identities!**: Use algebra and the identities are already known to you. Look to combine multiple fractions into one with a common denominator.



Using Addition and Subtraction Formulas

Prove the following identity: $\cos\left(\frac{\pi}{2} - u\right) = \sin u$

$$\text{LHS} = \cos\left(\frac{\pi}{2} - u\right)$$
$$= \cos\left(\frac{\pi}{2}\right)\cos u + \sin\left(\frac{\pi}{2}\right)\sin u$$
$$= \cancel{0 \cdot \cos u} + (1) \cdot \sin u$$
$$= \sin u = \text{RHS}$$

✓

Homework Problems

Prove the following identity:

19. $\sec\left(\frac{\pi}{2} - u\right) = \csc u$

$$= \frac{1}{\cos\left(\frac{\pi}{2} - u\right)} = \frac{1}{\sin u}$$

RHS

$$= \frac{1}{\cos\frac{\pi}{2}\cos u + \sin\frac{\pi}{2}\sin u} = \frac{1}{\sin u}$$

$$= \frac{1}{0 + \sin u}$$

$$= \frac{1}{\sin u}$$

✓

Example

$\frac{\pi}{4} = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right).$

Verify the following identity: $\frac{1+\tan x}{1-\tan x} = \tan\left(\frac{\pi}{4} + x\right)$

$$RHS = \tan\left(\frac{\pi}{4} + x\right) = \frac{\tan\frac{\pi}{4} + \tan x}{1 - \tan\left(\frac{\pi}{4}\right)\tan x}$$

$$= \frac{1 + \tan x}{1 - (1 \cdot \tan x)} = \frac{1 + \tan x}{1 - \tan x} = LHS$$

✓

Homework Problems

Verify the identity.

32. $\cos(x + y) + \cos(x - y) = 2\cos x \cos y$

LHS $= \cos(x+y) + \cos(x-y)$

$= \cos x \cos y - \sin x \sin y + \cos x \cos y + \sin x \sin y$

$= 2\cos x \cos y$ ✓

Homework Problems

Verify the identity.

34. $\cot(x+y) = \frac{(\cot x \cot y)-1}{\cot x+\cot y}$

LHS: $\frac{1}{\tan(x+y)} = \frac{1}{\frac{\tan x+\tan y}{1-\tan x\tan y}}$

$= \frac{1-\tan x\tan y}{\tan x+\tan y}$.

RHS: $\frac{\left(\frac{1}{\tan x}\cdot\frac{1}{\tan y}\right)-1}{\frac{1}{\tan y}+\frac{1}{\tan x}} = \frac{\frac{1-\tan x\tan y}{\tan x\tan y}}{\frac{\tan y+\tan x}{\tan x\tan y}}$

$= \frac{1-\tan x\tan y}{\tan x\tan y}\cdot\frac{\tan x\tan y}{\tan y+\tan x}$.

$= \frac{1-\tan x\tan y}{\tan x+\tan y}$. ✓

$(x+y)(x-y)$
$= x^2 - y^2$

Homework Problems

Verify the identity.

38. $\cos(x+y)\cos(x-y) = \cos^2 x - \sin^2 y$

$LHS = (\cos x \cos y - \sin x \sin y)(\cos x \cos y + \sin x \sin y)$

$= \cos^2 x \cos^2 y - \sin^2 x \sin^2 y$

$= \cos^2 x (1 - \sin^2 y) - (1 - \cos^2 x)\sin^2 y$

$= \cos^2 x - \cos^2 x \sin^2 y - (\sin^2 y - \cos^2 x \sin^2 y)$

$= \cos^2 x - \cos^2 x \sin^2 y - \sin^2 y + \cos^2 x \sin^2 y$

$= \cos^2 x - \sin^2 y = RHS$

✓

Homework 2/28

TB pg. 539-540 #19, 21, 27, 31, 32, 33, 34, 35, 37