Warm Up 2/21

$\boxed{1}$

Simplify the trigonometric expression.

1. $\cancel{\dfrac{\sec x - \cos x}{\tan x}}$

$\sin x$

2. $\dfrac{\sin x}{\csc x} + \dfrac{\cos x}{\sec x}$

$= \dfrac{\sin x}{\frac{1}{\sin x}} + \dfrac{\cos x}{\frac{1}{\cos x}}$

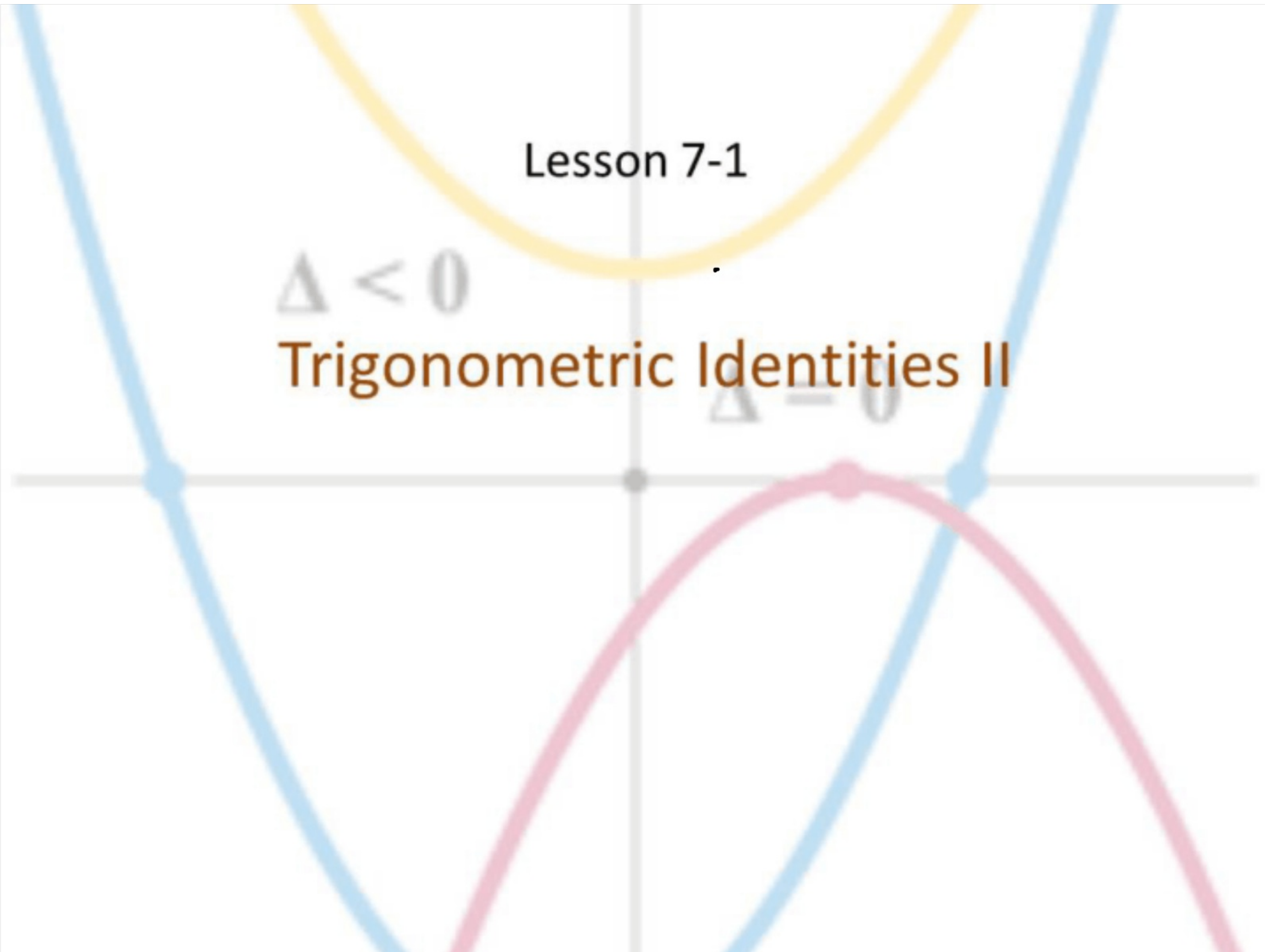
$= \dfrac{\sin x}{1}\left(\dfrac{\sin x}{1}\right) + \dfrac{\cos x}{1}\left(\dfrac{\cos x}{1}\right)$

$= \sin^2 x + \cos^2 x$

$= \boxed{1}$

Lesson 7-1

Trigonometric Identities II



Objective

Students will…

- Be able to prove or verify trigonometric identities.
- Be able to simplify expressions using trigonometric substitution.

Trigonometric Identities

Before we get any deep into trig analysis, we must first recall some of the basic trigonometric identities and definitions. Primarily,

$\csc x = \frac{1}{\sin x}$ $\qquad$ $\sec x = \frac{1}{\cos x}$ $\qquad$ $\cot x = \frac{1}{\tan x}$

$\tan x = \frac{\sin x}{\cos x}$ $\qquad$ $\cot x = \frac{\cos x}{\sin x}$

Pythagorean Identity: $\sin^2 x + \cos^2 x = 1$

From this, we also get:

$\sin^2 x = 1 - \cos^2 x$ and $\cos^2 x = 1 - \sin^2 x$

$\tan^2 x + 1 = \sec^2 x$ and $1 + \cot^2 x = \csc^2 x$

$\tan^2 x = \sec^2 x - 1$ and $\cot^2 x = \csc^2 x - 1$

Methods for Proving Identities

One of the main components of trig analysis is to prove identities. There are two different methods for proving identities: LHS "=" RHS.

I. Rewrite **one** of the sides to match the other side.

Ex.

LHS $x + 3 = 6\left(\frac{1}{6}x + \frac{11}{6} - \frac{13}{6} + \frac{5}{6}\right)$ RHS.

$= x + 11 - 13 + 5$

$\boxed{\text{LHS} = x + 3}$ ✓

II. Modify **both** sides until they are the same.

Ex.

$3(2x - 1) = 2x + 2\left(2x - \frac{3}{2}\right)$

$6x - 3 = 2x + 4x - 3$

$6x - 3 = 6x - 3$ ✓

Guidelines for Proving Identities

Furthermore, we have some guidelines/tips for proving identities.

1. **<u>Focus on the fractions</u>**: More often than not, identity proofs are more easily done when you work with the side that involves a fraction.

2. **<u>Pick the more "complicated" side</u>**: It's easier to modify the sides that has less sines or cosines. Generally, rewriting everything as sine or cosine can help you when you are "stuck."

3. **<u>Use the Known Identities!</u>**: Use <u>algebra</u> and the identities are already known to you. Look to combine multiple fractions into one with a common denominator.

Example

LHS RHS.

Prove/Verify the identity: $\cos\theta\,(\sec\theta - \cos\theta) = \sin^2\theta$

LHS: $\cos\theta(\sec\theta - \cos\theta)$

$= \frac{\cos\theta}{1}\left(\frac{1}{\cos\theta} - \cos\theta\right)$

$= 1 - \cos^2\theta$

$= \sin^2\theta = \text{RHS}$

$\cos\theta(\sec\theta - \cos\theta) = \sin^2\theta$

$\sin^2\theta$

$\sin^2\theta$

$\sin^2\theta = \sin^2\theta$

$\therefore$

$\cos\theta(\underbrace{\sec\theta}_{\frac{1}{\cos}} - \cos\theta) = \sin^2\theta$

$1 - \cos^2\theta = 1 - \cos^2\theta$

Example

Prove/Verify the identity: $\cos x \tan x = \sin x$

LHS: $\cos x \tan x$

$= \frac{\cancel{\cos x}}{1}\left(\frac{\sin x}{\cancel{\cos x}}\right)$

$= \sin x = \text{RHS}$

✓

Homework Problems

Verify the identity:

31. $\sin B + \cos B \cot B = \csc B$

LHS: $\sin B + \cos B \cot B$

$= \sin B + \frac{\cos B}{1}\left(\frac{\cos B}{\sin B}\right)$

$= \frac{\sin B}{1} \cdot \frac{\sin B}{\sin B} + \frac{\cos^2 B}{\sin B}$

$= \frac{\sin^2 B}{\sin B} + \frac{\cos^2 B}{\sin B} = \frac{\sin^2 B + \cos^2 B}{\sin B} = \frac{1}{\sin B}$

$= \csc B = \text{RHS}$

$x^2 - 9$

$(x+3)(x-3)$

Example

$(x-1)(x+1)$

$x^2 - 1$

$\frac{1}{(x-1)} - \frac{1}{(x+1)}$

Prove/Verify the identity: $2\tan x \sec x = \frac{1}{1-\sin x} - \frac{1}{1+\sin x}$

LHS: $2 \tan x \sec x$

$= \frac{2}{1}\left(\frac{\sin x}{\cos x}\right)\left(\frac{1}{\cos x}\right)$

$= \boxed{\frac{2\sin x}{\cos^2 x}}$

RHS: $\frac{1}{1-\sin x} - \frac{1}{1+\sin x} = \frac{1+\sin x}{(1-\sin x)(1+\sin x)} - \frac{1-\sin x}{(1+\sin x)(1-\sin x)}$

$= \frac{1+\sin x - (1-\sin x)}{(1+\sin x)(1-\sin x)} = \frac{\cancel{1}+\sin x - \cancel{1}+\sin x}{(1+\sin x)(1-\sin x)}$

$= \frac{2\sin x}{(1+\sin x)(1-\sin x)} = \frac{2\sin x}{1 \cancel{-\sin x + \sin x} - \sin^2 x}$

$= \frac{2\sin x}{1-\sin^2 x} = \boxed{\frac{2\sin x}{\cos^2 x}}$

Homework Problems

Verify the identity :

29. $\frac{\tan y}{\csc y} = \sec y - \cos y$

LHS: $\frac{\tan y}{\csc y} = \frac{\frac{\sin y}{\cos y}}{\frac{1}{\sin y}}$

$= \frac{\sin y}{\cos y} \cdot \frac{\sin y}{1} = \frac{\sin^2 y}{\cos y}$

RHS: $\sec y - \cos y$.

$\frac{1}{\cos y} - \frac{\cos y \cdot \cos y}{1 \cdot \cos y}$

$= \frac{1}{\cos y} - \frac{\cos^2 y}{\cos y} = \frac{1-\cos^2 y}{\cos y}$.

$= \frac{\sin^2 y}{\cos y}$

✓

Example

Prove/Verify the identity: $\tan\theta + \cot\theta = \sec\theta\csc\theta$

$LHS = \tan\theta + \cot\theta$

$\frac{\sin\theta}{\sin\theta} \cdot \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} \cdot \frac{\cos\theta}{\cos\theta}$

$= \frac{\sin^2\theta}{\sin\theta\cos\theta} + \frac{\cos^2\theta}{\sin\theta\cos\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \frac{1}{\cos\theta\sin\theta}$

$RHS = \sec\theta\csc\theta$

$= \frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta}$

$= \frac{1}{\cos\theta\sin\theta}$

✓

Homework Problems

Verify the identity :

35. $\tan\theta + \cot\theta = \sec\theta\csc\theta$

$x^2-9=(x+3)(x-3)$

Example

$\frac{2}{1-i}\cdot\left(\frac{1+i}{1+i}\right)$

Prove/Verify the identity: $\frac{\cos u}{1-\sin u}=\sec u+\tan u$

LHS $=\frac{\cos u}{1-\sin u}\cdot\frac{1+\sin u}{1+\sin u}$

$=\frac{\cos u(1+\sin u)}{(1-\sin u)(1+\sin u)}=\frac{\cos u(1+\sin u)}{\cancel{1+\sin u-\sin u}-\sin^2 u}$

$=\frac{\cos u(1+\sin u)}{1-\sin^2 u}=\frac{\cancel{\cos u}(1+\sin u)}{\cos^2 u}$

$=\frac{1+\sin u}{\cos u}$

RHS $=\sec u+\tan u$.

$=\frac{1}{\cos u}+\frac{\sin u}{\cos u}=\frac{1+\sin u}{\cos u}$.

Homework Problems

Verify the identity :

51. $\frac{1-\cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1+\cos\alpha}$

$$LHS = \frac{(1-\cos a)}{\sin a} \cdot \frac{(1+\cos a)}{1+\cos a}$$

$$= \frac{1-\cos^2 a}{\sin a(1+\cos a)}$$

$$= \frac{\sin^2 a}{\cancel{\sin a}(1+\cos a)}$$

$$= \frac{\sin a}{1+\cos a} = RHS \checkmark$$

Homework Problems

$xy + xy = 2xy.$

Verify the identity :

85. $\frac{1+\sin x}{1-\sin x} = (\tan x + \sec x)^2$

$LHS = \frac{(1+\sin x)}{1-\sin x} \cdot \frac{(1+\sin x)}{1+\sin x}$

$= \frac{1+\sin x+\sin x+\sin^2 x}{1-\sin^2 x}$

$= \frac{1+2\sin x+\sin^2 x}{\cos^2 x}$

$RHS = (\tan x+\sec x)^2 = (\tan x+\sec x)(\tan x$

$= \tan^2 x + \tan x \sec x + \sec x \tan x + \sec^2 x$

$= \tan^2 x + 2\tan x \sec x + \sec^2 x.$

$= \frac{\sin^2 x}{\cos^2 x} + \frac{2}{1}\left(\frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}\right) + \frac{1}{\cos^2 x}.$

$= \frac{\sin^2 x}{\cos^2 x} + \frac{2\sin x}{\cos^2 x} + \frac{1}{\cos^2 x}$

$= \frac{\sin^2 x + 2\sin x + 1}{\cos^2 x}.$

√

Homework 2/21

TB pg. 533 #29, 31, 35, 37, 41, 49, 51, 59, 73, 79, 85