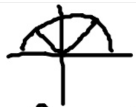
Warm Up 2/10



Use the Law of Sines to solve the triangle.

$a = 26, c = 15, \angle C = 29°$

① $\cancel{(26)}\frac{\sin A}{\cancel{26}} = \frac{\sin 29°}{15}(26) =$

$\cancel{\sin^{-1}}(\cancel{\sin})A = \sin^{-1}(0.84)$

$A = 57.1°$

$B = 93.9°$

$\frac{\sin 93.9}{b} = \frac{\sin 29}{15} = 0.0323$

$b = 30.9$

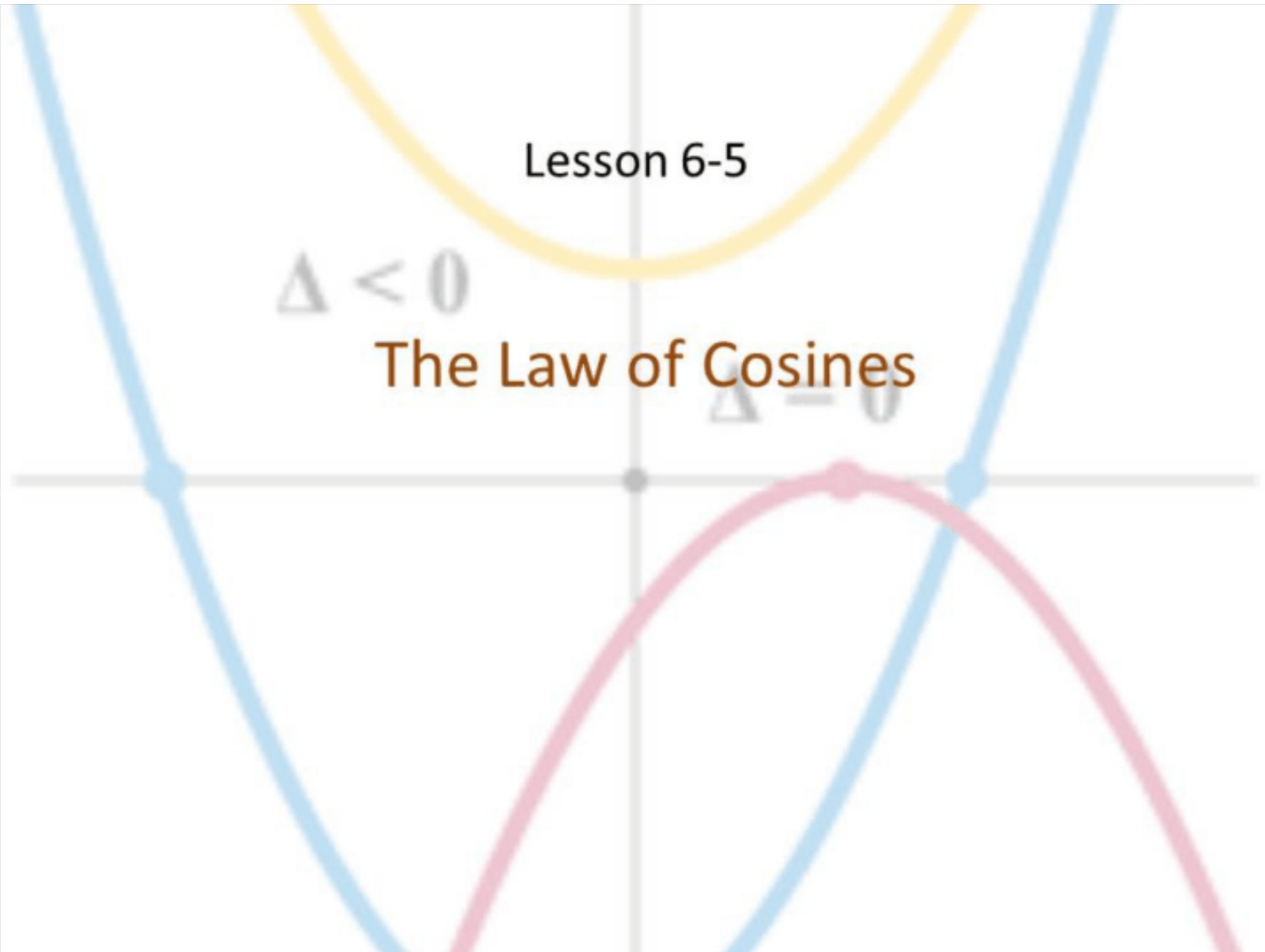
$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

② $180 - 57.1 = 122.9 = A$

$B = 28.1$

$\frac{\sin 28.1}{b} = \frac{\sin 29}{15} = 0.0323$

$b = 14.6$


Lesson 6-5
Δ < 0
The Law of Cosines
Δ = 0

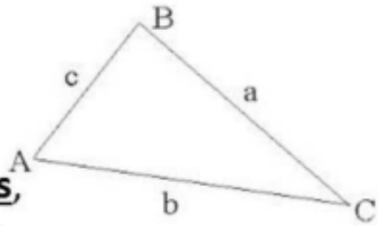
Objective

Students will…

- Be able to know what Law of Cosines is.
- Be able to apply the Law of Cosines to solve for missing sides or angles.

Triangles

We’ve been studying the trigonometric ratios involving right triangles. Trigonometry can also be used for **non**-right triangles. First thing we need to do is to be consistent with our notations.



Consider the triangle ΔABC shown on the right.
The uppercase letters A, B, C represent the **vertices**, or the **angles** of the triangle, while the lower case letters a, b, c represent the sides. For ease, the angles will always be labeled by uppercase letters, while the side **opposite** to each angle, will always be labeled with the lowercase letter of the opposite angle.

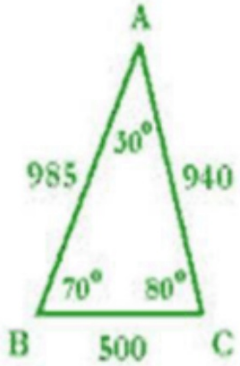
So, from our picture, we see that a is the side opposite to A, while b is the side opposite to B and c is the side opposite to C.

Law of Cosines

There exists another important law regarding triangles (not just right triangles).

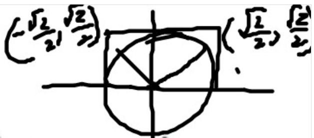
Law of Cosines- In any triangle, say, ΔABC, we have:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$



For the ΔABC to the left, we have...

$$500^2 = 940^2 + 985^2 - 2(940)(985)\cos 30°$$

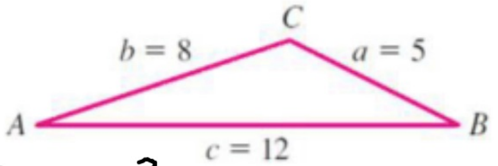


Example

So we can apply the Law of Cosines to solve for missing sides or angles.

(Important: Make sure your calculator is in the right mode!)

Find the angles of the triangle.



$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$8^2 = 5^2 + 12^2 - 2(5)(12)\cos B$$

$$64 = \cancel{25} + \cancel{144} - 120\cos B$$

$$-25 \quad -144 \quad -25 \quad -144$$

$$\frac{-105}{-120} = \frac{\cancel{-120}\cos B}{\cancel{-120}}$$

$$\cos^{-1} = \cos B$$

$$B \approx 29^\circ$$

$$\boxed{A = 17.6^\circ}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$12^2 = 5^2 + 8^2 - 2(5)(8)\cos C$$

$$144 = 25 + 64 - 80\cos C$$

$$-25 \quad -64 \quad -25 \quad -64$$

$$\frac{55}{-80} = \frac{\cancel{-80}\cos C}{\cancel{-80}}$$

$$-0.6875 = \cos C$$

$$C = 133.4$$

Example

Solve ΔABC, where $\angle A = 46.5°$, $b = 10.5$, and $c = 18$

$a^2 = b^2 + c^2 - 2bc \cos A$

$a^2 = 10.5^2 + 18^2 - 2(10.5)(18) \cos 46.5°$

$a^2 = 110.25 + 324 - 378 \cos 46.5$

$\sqrt{a^2} = \sqrt{174.1}$

$\boxed{a = 13.2}$

$b^2 = a^2 + c^2 - 2ac \cos B$

Heron's (Area) Formula

An interesting application of the Law of Cosines involves a formula for finding the area of a triangle from the lengths of its three sides. We won't derive the formula here for time's sake. (see textbook)

<u>Heron's Formula</u>- For ΔABC the area $\mathcal{A} = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$, which is the **<u>semiperimeter</u>** (half perimeter).

Ex. Find the area of a triangle with given side lengths:
$a = 280$, $b = 125$, and $c = 315$

$S = \frac{1}{2}(280+125+315) = 360$

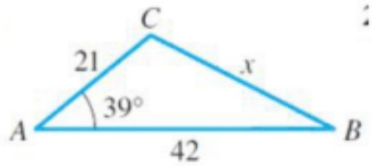
$\mathcal{A} = \sqrt{360(360-280)(360-125)(360-315)}$

$\boxed{\mathcal{A} = 17{,}451.6\ u^2}$

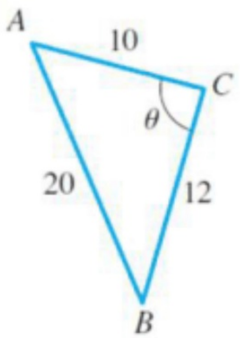
Homework Problems

Use the Law of Cosines to determine the indicated side x or angle θ.

1.



8.



Homework Problems

Solve the triangle.

11. $a = 3, b = 4, \angle C = 53°$

Homework Problems

Solve the triangle. Law of sine.

17. $a = 50, b = 65, \angle A = 55°$ $c = ?$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$(65)\frac{\sin 55°}{50} = \frac{\sin B}{\cancel{65}}\cancel{(65)}$$

$$1 < 1.065 = \sin B$$ ← Impossible

No sol.

Homework Problems

Find the area of the triangle.

27. $a = 9, b = 12, c = 15$

Homework 2/10

TB pg. 513 #1, 3, 5, 8, 11-17 (odd), 27, 29

13) $a=20,\ b=25,\ c=22.$

$$b^2=a^2+c^2-2ac\cos B.$$

$$25^2=20^2+22^2-2(20)(22)\cos B$$

$$625=400+484-880\cos B$$

$$-480\ -484\quad -400\ -484$$

$$\frac{-259}{-880}=\frac{-880\cos B}{-880}$$

$$\cos^{-1}(0.294)=\cos^{-1}\cos B$$

$$\boxed{B=72.9^\circ}$$

$$c^2=a^2+b^2-2ab\cos C.$$

$$22^2=20^2+25^2-2(20)(25)\cos C.$$

$$484=400+625-1000\cos C.$$

$$-400\ -625\quad -400\ -625$$

$$\frac{-541}{-1000}=\frac{-1000\cos C}{-1000}.$$

$$\cos^{-1}(+0.541)=\cos^{-1}\cos C$$

$$\boxed{C=57.2^\circ}$$

$$A=180-B-C$$

$$\boxed{A=49.9^\circ}$$