Warm Up 2/6

Solve the following proportions.

1. $\frac{5}{y} = \frac{45}{63}$

2. $\frac{2y}{9} = \frac{8}{4y}$

3. $\frac{2}{x-3} = \frac{8}{3x-3}$


Lesson 6-4
$\Delta < 0$
The Law of Sines

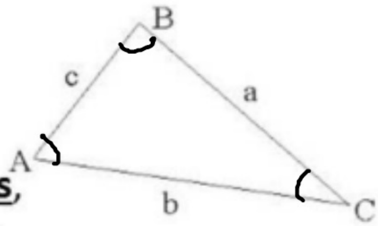
Objective

Students will…

- Be able to know what Law of Sines is.
- Be able to apply the Law of Sines to solve for missing sides or angles.

Triangles

We've been studying the trigonometric ratios involving right triangles. Trigonometry can also be used for **non**-right triangles. First thing we need to do is to be consistent with our notations.



Consider the triangle ΔABC shown on the right.
The uppercase letters A, B, C represent the **vertices**, or the **angles** of the triangle, while the lower case letters a, b, c represent the sides. For ease, the angles will always be labeled by uppercase letters, while the side **opposite** to each angle, will always be labeled with the lowercase letter of the opposite angle.

So, from our picture, we see that a is the side opposite to A, while b is the side opposite to B and c is the side opposite to C.

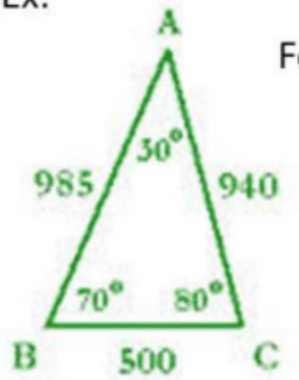
Law of Sines

There exists and important law regarding triangles (not just right triangles) derived from its **area formula**.

Law of Sines- For any triangle the lengths of its sides are proportional to the sines of the corresponding opposite angles. Namely, for ΔABC:

$$\boxed{\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}}$$

Ex.



For the ΔABC to the left, we have...

$$\frac{\sin 30^\circ}{500} = \frac{\sin 70^\circ}{940} = \frac{\sin 80^\circ}{985}.$$

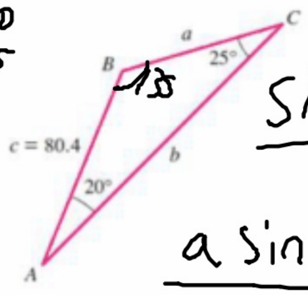
Example

So we can apply the Law of Sines to solve for missing sides or angles.

(<u>Important</u>: Make sure your calculator is in the right mode!)

Find a and b.

180
45



20
$+25$
45

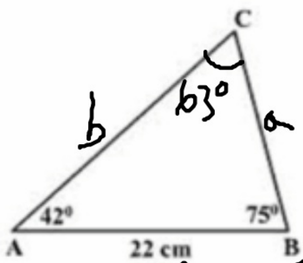
$$\frac{\sin 20^\circ}{a} = \frac{\sin 25^\circ}{80.4}$$

$$\frac{a \cancel{\sin 25^\circ}}{\cancel{\sin 25^\circ}} = \frac{27.5}{\sin 25^\circ}$$

$$\frac{\sin 135^\circ}{b} = \frac{\sin 25}{80.4}$$

$$\boxed{b = 134.6}$$

Solve the triangle (i.e. find all missing sides and angles).



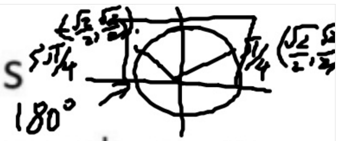
$$\frac{\sin 42^\circ}{a} = \frac{\sin 63^\circ}{22}$$

$$a = \boxed{16.5}$$

$$\frac{\sin 75^\circ}{b} = \frac{\sin 63^\circ}{22}$$

$$b = \boxed{23.8}$$

Ambiguous Cases: Two solutions



The two previous examples had **two** known angles. There may be a case where we might only have **one** known angle, but with two known sides. In either case, Law of Sines can be applied to solve the triangle. However, one important thing to bear mind here is the fact there may be more than one, or even **no** correct answer when the Law of Sines is applied with one known angle and two known sides. Consider the following: *Solve triangle* ΔABC *if* $\angle A = 43.1°$, $a = 186.2$, *and* $b = 248.6$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$(248.6)\frac{\sin 43.1}{186.2} = \frac{\sin B}{\cancel{248.6}}\cancel{(248.6)}$$

$$\sin^{-1}(0.9123) = \cancel{\sin}\, B$$

$$65.8 = B$$

0.0037

(1) $B = \boxed{65.8°}$

$C = 180 - 43.1 - 65.8$

$= \boxed{71.1°}$

$$\frac{\sin 43.1}{186.2} = \frac{\sin 71.1}{c}$$

$$\boxed{257.8} = c$$

0.0037

(2) $180 - 65.8 = \boxed{114.2} = B$

$C = 180 - 43.1 - 114.2 = \boxed{22.7}$

$$\frac{\sin 43.1}{186.2} = \frac{\sin 22.7}{c}$$

$$\boxed{104.3} = c$$

Ambiguous Cases: One solution

Now consider: *Solve triangle ΔABC if $\angle A = 45°$, $a = 7\sqrt{2}$, and $b = 7$*

$$(\cancel{7})\frac{\sin 45}{\cancel{7}\sqrt{2}} = \frac{\sin B}{\cancel{7}}(\cancel{7})$$

① $$\frac{\sin 45}{\sqrt{2}} = \sin B$$

$$\sin^{-1}(0.5) = \sin^{-1}\sin B$$

$$\boxed{C = 105°} \quad \boxed{B = 30°}$$

$$\frac{\sin C}{c} = \frac{\sin B}{b}.$$

$$\frac{\sin 105}{c} = \frac{\sin 30}{7} = 0.0714.$$

$$\boxed{c = 13.5}$$

② $180 - 30 = 150 = B.$ Impossible.

$150 + 45 = 195 > 180.$

Only 1 solution.

Ambiguous Cases: No solution

Now consider: *Solve triangle ΔABC if $\angle A = 42°$, $a = 70$, and $b = 122$*

$$(122)\frac{\sin 42}{70} = \frac{\sin B}{\cancel{122}}\cancel{(122)}$$

$$\sin^{-1}(1.166) = \cancel{\sin} B$$

$\sin^{-1}$

Error

No sol.

Sine is never greater tha 1

General Guideline: Law of Sines

Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Two angles and **one side** known: Only **1** possible outcome.

One angle and **two sides** known:

Case I- One outcome (angle measure of triangles cannot exceed 180)

Case II- Two possible outcomes

Case III- No possible outcome (sine of an angle cannot be greater than 1)

ex. $\sin A = 1.239$ $\rightarrow$ no possible solution.

Homework Problems

Solve each triangle using the Law of Sines.

11. $\angle A = 50°, \angle B = 68°, c = 230$, $\angle C = 62°$

$$\frac{\sin 50°}{a} = \frac{\sin 62}{230} = 0.0038$$

$$\boxed{a = 199.5}$$

201.6

$$\frac{\sin 68}{b} = \frac{\sin 62}{230} = 0.0038$$

$$\boxed{b = 241}$$

Homework Problems

Solve each triangle using the Law of Sines.

19. $a = 20, c = 45, \angle A = 125°$

$$(45)\frac{\sin 125}{20} = \frac{\sin C}{\cancel{45}}\cancel{(45)}$$

$$1.843 = \sin C$$

No Sol

Homework Problems

Solve each triangle using the Law of Sines.

21. $b = 25$, $c = 30$, $\angle B = 25°$

$$(30)\frac{\sin 25}{25} = \frac{\sin C}{\cancel{30}}\cancel{(30)}$$

$$\sin^{-1}(0.507) = \cancel{\sin}\ C$$

$\sin^{-1}$

$$30.5 = C$$

$A = 124.5$

$a =$

$C = 149.5$

$180 - 30.5 = \cancel{155}$

$A =$

$180 - 149.5 + 25 = 5.5$

$a =$

Homework 2/6

TB pg. 506 #1, 3, 5, 11, 15, 17, 19, 21, 23, 25