

Warm Up 2/5

Convert the following degree measures into radian.

1. 67°

$$\frac{67}{1} \cdot \frac{\pi}{180} = \boxed{\frac{67\pi}{180}}$$

2. 22°

$$\cancel{11} \frac{\cancel{22}}{1} \cdot \frac{\pi}{\cancel{180}} = \boxed{\frac{11\pi}{90}}$$

3. 299°

4. -112°

$$\frac{-\cancel{28}}{\cancel{112}} \cdot \frac{\pi}{\cancel{180}} = \boxed{\frac{-28\pi}{45}}$$

5. 0.4°

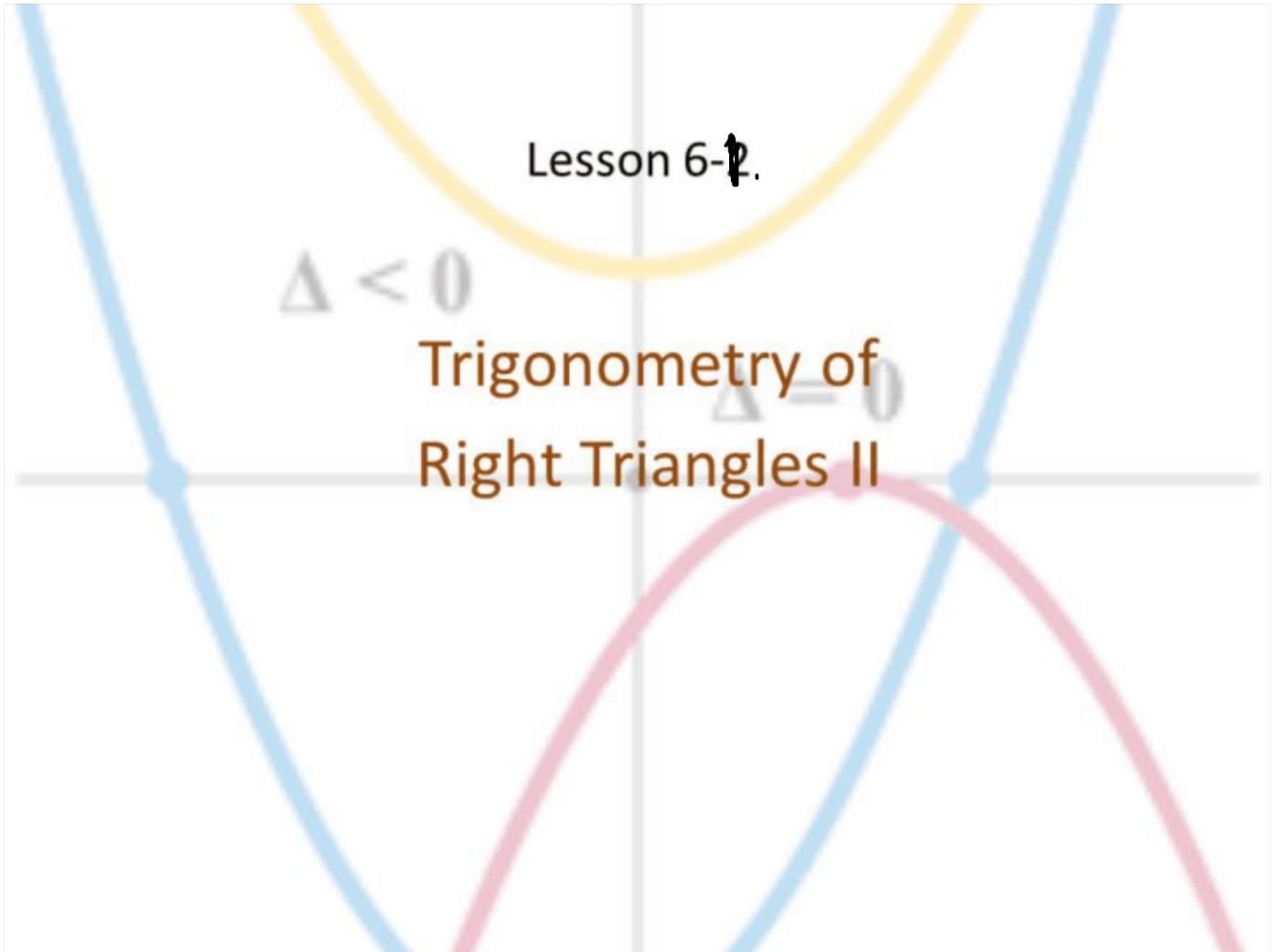
$$\frac{0.4}{1} \cdot \frac{\pi}{180} = \frac{4}{10} \cdot \frac{\pi}{180} = \frac{\cancel{1} \cancel{2} \pi}{5 \cdot \cancel{180}} = \boxed{\frac{\pi}{450}}$$

Lesson 6-1.

$\Delta < 0$

Trigonometry of
Right Triangles II

$\Delta = 0$



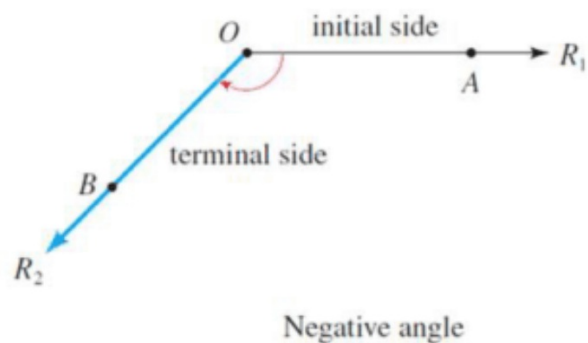
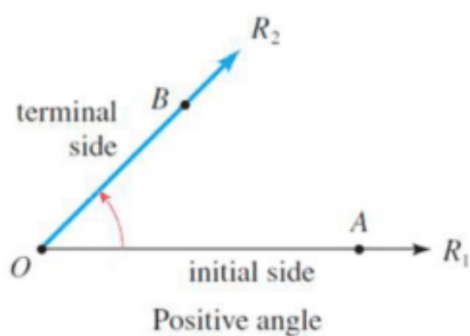
Objective

Students will...

- Be able to calculate the length of a circular arc and circular sector area, given the radius of the circle and the angle measure.
- Be able to differentiate between angular and linear speed and compute them.

Angles

In trigonometry, however, angles are viewed as **rotations** of **one** line. In other words, angle measurement represents the **distance** travelled, or rotated. The beginning, or the stationary, position is known as the **initial side**, while the line at its finishing position is known as the **terminal side**. In this case, rotating **counter-clockwise** is **positive**, while rotating **clockwise** is **negative**.





Circular Arc Length



In our last lesson, we learned that radian represented the angle measurement of the rotation, or the distance travelled. In this lesson, we now want to calculate the linear length of this distance. This is known as the arc length. Even though an arc is a part of a circle, we say linear length, since we can always picture cutting this arc out and laying it flat, or straight. We would then be able to measure the length with a ruler, for example. Following is the formula for measuring the arc length, s , with radius, r and radian angle measurement θ :

$$s = r\theta$$

We can then modify this equation and get a very important formula:

$$\theta = \frac{s}{r}$$

One thing to keep in mind is we always need to use radians.

Example

Find the length of an arc of a circle with radius 10m that subtends a central angle of 30° .

$$\frac{\cancel{30}}{1} \cdot \frac{\pi}{\cancel{180}} = \frac{\pi}{6} = \theta$$

$$S = \overset{r}{10} \cdot \theta = \frac{10}{1} \cdot \left(\frac{\pi}{6}\right) = \boxed{\frac{5\pi}{3}}$$

Find the length of an arc of a circle with radius 21m that subtends a central angle of 15° .

$$\frac{\cancel{15}}{1} \cdot \frac{\pi}{\cancel{180}} = \frac{\pi}{12} = \theta$$

$$S = \overset{r}{21} \cdot \theta = \frac{21}{1} \cdot \left(\frac{\pi}{12}\right) = \boxed{\frac{7\pi}{4}}$$

A central angle θ in a circle of radius 4m is subtended by an arc of length $s = 6$ m. Find the measure of θ in radians.

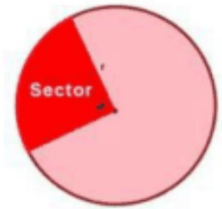
$$s = r\theta$$
$$\theta = \frac{s}{r}$$
$$\theta = \frac{6}{4} = \frac{3}{2}$$

A central angle θ in a circle of radius 9m is subtended by an arc of length 12m. Find the measure of θ in radians.

Area of a Circular Sector

We can also find the area of a circular sector by any given central angle θ . The section in red is the circular sector.

Combining with the area formula of a circle: $A = \pi r^2$, we get the following formula for finding the area of a given circular sector.



$$A = \frac{1}{2} r^2 \theta$$

Ex. Find the area of a sector of a circle with central angle 60° if the radius of the circle is 3m.

$$\frac{1}{3} \cdot \frac{\pi}{3} = \frac{\pi}{9} = \theta$$

$$A = \frac{1}{2} (3^2) \left(\frac{\pi}{3} \right) = \frac{9}{2} \left(\frac{\pi}{3} \right) = \frac{3\pi}{2}$$

Circular Motion

The last thing regarding the angular rotation is the concept of **circular motion**, which simply describes an object moving in circles. There are two ways to describe this type of motion: **linear** and **angular** speed.

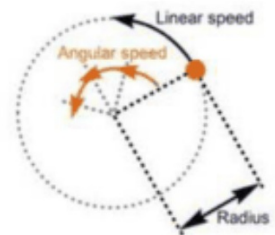


Linear speed describes the **explicit** distance travelled over **time** (i.e. how fast the object is traveling along the circle). The unit is $\frac{\text{distance (m,miles,etc.)}}{\text{time}}$

Linear Speed (v): $v = \frac{s}{t}$, where s is the **arc length**.

Angular speed, on the other hand, describes the **angular change** over time (i.e. how fast the angle is changing). The unit is $\frac{\text{angle (rad or deg)}}{\text{time}}$

Angular Speed (ω): $\omega = \frac{\theta}{t}$



Example

A boy rotates a stone in a 3-ft-long sling at the rate of 15 revolutions every 10 seconds. Find the angular and linear speeds of the stone.

$$v = \frac{s}{t} = \frac{90\pi}{10} = 9\pi \text{ ft/sec}$$

$$s = r\theta = 3(3\pi) = 9\pi$$

$$15 \cdot 2\pi = \frac{30\pi}{10} = 3\pi \text{ /sec}$$

$$\omega = \frac{\theta}{t} = 3\pi \text{ /sec}$$

A disk with a 12-inch diameter spins at the rate of 45 revolutions per minute. Find the angular and linear velocities of a point at the edge of the disk in radians per second and inches per second, respectively.

Linear and Angular Speed

It turns out that there is a way to take any angular speed and find its corresponding linear speed. With v being the linear speed, and ω being the angular speed, with radius r we have the following:

$$v = r\omega$$

Ex. A woman is riding a bicycle whose wheels are 26 inches in diameter. If the wheels rotate at 125 rpm (revolutions per minute), find the speed at which she is traveling.

$$\omega = 125 \cdot 2\pi = 250\pi \text{ rad/min}$$

$$\begin{aligned} v &= 13(250\pi) \\ &= 3250\pi \text{ in/min.} \end{aligned}$$

A woman is riding a bicycle whose wheels are 30 inches in diameter. If the wheels rotate at 150 rpm (revolutions per minute), find the speed at which she is traveling.

Homework 2/5

TB pg. 475 #49-51, 53, 59, 79, 81