Warm Up 2/4

$= \boxed{5}.7\ldots$ 0.72 ----

1. Find the remainder: $\frac{128475}{22445}$ (Yeah, I would use a calculator here)

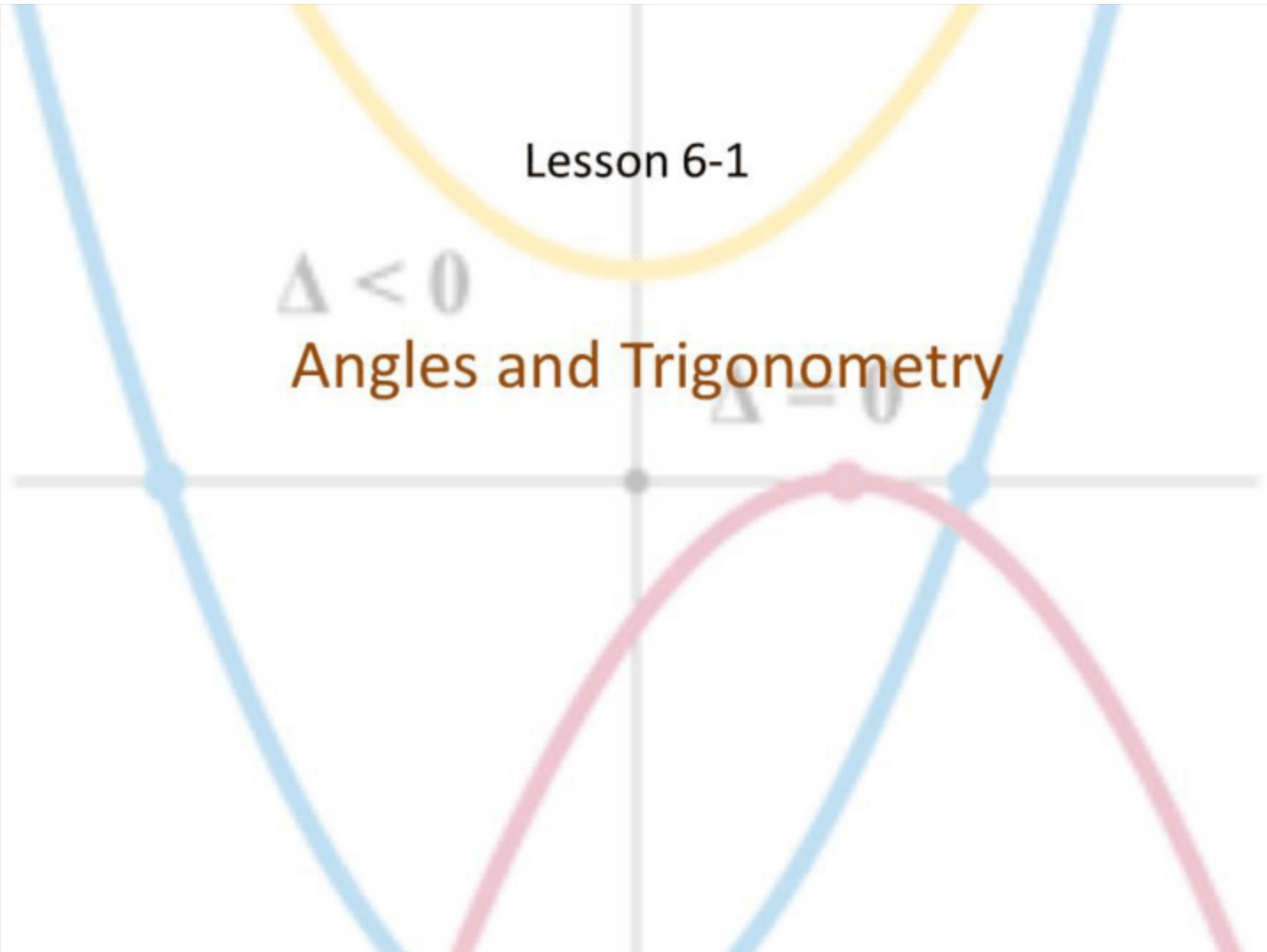
(hint: first figure out how many times 22445 can multiply into 128675)

$= \boxed{16250}$

2. How many times does 2π go (multiply) into 100? (Again, calculator!)

$100 \div (2\pi)$ $= \boxed{15}$

$\cancel{(100 \div 2)\pi}$


Lesson 6-1
Δ < 0
Angles and Trigonometry

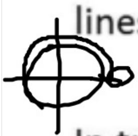
Objective

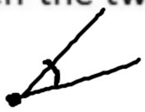
Students will…

- Be able to understand radian angle measure as distance traveled or rotated.
- Be able to convert radian into degrees and vice-versa.
- Be able to find coterminal angles.

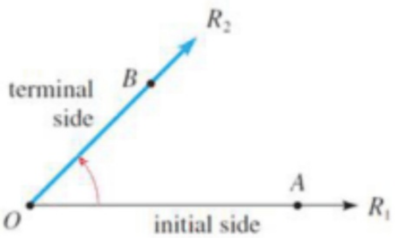
Angles

Back in geometry, we studied angles and their measurements. We mainly viewed angle measurements as the "space" between the two lines sharing a same point, namely, the vertex.

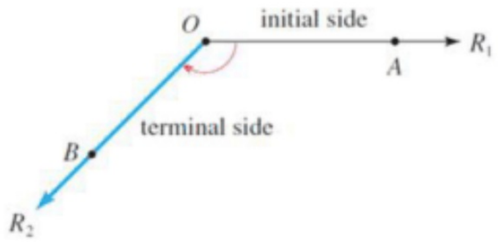




In trigonometry, however, angles are viewed as **rotations** of **one** line. In other words, angle measurement represents the **distance** travelled, or rotated. The beginning, or the stationary, position is known as the **initial side**, while the line at its finishing position is known as the **terminal side**. In this case, rotating **counter-clockwise** is positive, while rotating **clockwise** is negative.



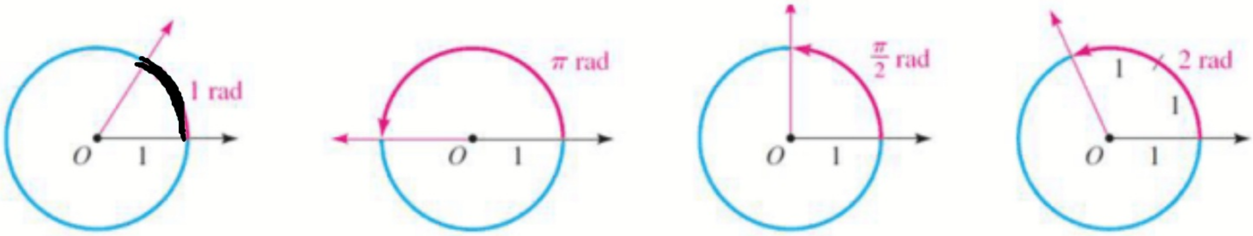
Positive angle



Negative angle

Radians

With that said, the amount of rotation, or the distance travelled in a **circular** motion is measured in **radians** (sometimes abbreviated **rad**). As we remember from the unit circle, one full counter-clockwise rotation is **2π** rad. Half-way around is **π** rad. See the figure below.



Degrees vs Radians

Of course, it's not hard to understand that, when measured in degrees, one full revolution around a circle is $\underline{\mathbf{360^\circ}}$. With that said, consider the following:

$$\frac{360^\circ}{2} = \frac{\cancel{2}\pi}{\cancel{2}} \text{ rad} \rightarrow \frac{\cancel{180}^\circ}{180} = \frac{\pi \text{ rad}}{180}$$

$$\frac{180^\circ}{\pi} = 1 \text{ rad.}$$

$$1^\circ = \frac{\pi}{180} \text{ rad.}$$

So, we conclude the following:

1. To convert from deg to rad, multiply by $\frac{\pi}{180}$
2. To convert from rad to deg, multiply by $\frac{180}{\pi}$

Examples

deg → rad.

1. Express 60° in radians

$$\frac{\cancel{60}^{1}}{1} \cdot \frac{\pi}{\cancel{180}_{3}} = \boxed{\frac{\pi}{3}}$$

rad → deg.

2. Express $\frac{\pi}{6}$ in degrees

$$\frac{\cancel{\pi}}{\cancel{6}_{1}} \cdot \frac{\cancel{180}^{30}}{\cancel{\pi}} = \boxed{30°}$$

3. Express 20° in radians

$$\frac{20}{1} \cdot \frac{\pi}{180} = \boxed{\frac{\pi}{9}}$$

4. Express $\frac{11\pi}{7}$ in degrees

$$\frac{11\cancel{\pi}}{7} \cdot \frac{180}{\cancel{\pi}} = \frac{1980}{7}$$

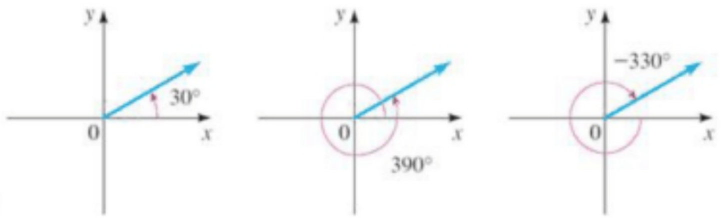
$$\approx \boxed{282.9°}$$

Coterminal Angles

Going back to the idea of angles moving in rotations, in any type of circular rotation, the line (or whatever object is moving) is bound to return to the **initial position** over and over again. Take a trivial example of 2π. We know that the angles measuring 0 rad and 2π rad are in the exact **same** position. Same goes for $4\pi = 8\pi = 22\pi = \cdots$

This can also be easily seen in degree measurements as well. A line rotating $30°$ counter-clockwise would be in the same position if it were to rotate $390°$, or going the opposite direction, $330°$ clockwise $(-330°)$.

These angles are known as **coterminal angles**, meaning that the angles have the same terminal, or **ending** position.



Finding Coterminal Angles

Now, finding coterminal angles is easy. No matter where the initial position is, if a line was to rotate **$\underline{360°}$** or **$\underline{2\pi}$** rad, it would naturally end up in the **<u>same</u>** position.

<u>Degrees</u>- When finding coterminal angles in degrees, we can simply add any **<u>multiples of 360</u>**.

<u>Radians</u>- When finding coterminal angles in radians, we can simply add any **<u>multiples of 2π.</u>**

Examples

1. Find one positive and one negative angle that is coterminal with 30°

$30^\circ + 360 = \boxed{390^\circ}$

$30^\circ + -360 = \boxed{-330^\circ}$

2. Find one positive and one negative angle that is coterminal with $\frac{\pi}{3}$ rad.

$\frac{\pi}{3} + \frac{2\pi \cdot 3}{1 \cdot 3} \quad \frac{6\pi}{3} = \boxed{\frac{7\pi}{3}}$ $\qquad$ $\frac{\pi}{3} - \frac{2\pi}{1} \quad \frac{6\pi}{3} = \boxed{-\frac{5\pi}{3}}$

3. Find one positive and one negative angle that is coterminal with 111°

$111 + 360 = \boxed{471^\circ}$ $\qquad$ $111 - 720 = -609$

$111 + 720 = \boxed{831^\circ}$ $\qquad$ $111 - 360 = \boxed{-249}$

4. Find one positive and one negative angle that is coterminal with $-\frac{7\pi}{2}$

$-\frac{7\pi}{2} + \frac{4\pi}{2} = \boxed{-\frac{3\pi}{2}} + \frac{4\pi}{2} = \boxed{\frac{\pi}{2}}$

5. Find an angle between 0 and 2π that is coterminal with 111π rad.

$\frac{111}{2} = 55$

$2\pi \cdot 55 = 110\pi$

$111\pi - 110\pi = \boxed{\pi}$.

6. Find an angle between $0°$ and $360°$ that is coterminal with $-3624°$.

$\frac{3624}{360} = \boxed{10}$

$+1$

11.

$360 \times 11 = 3960$

$-3624 + 3960 = \boxed{336°}$.

7. Find an angle with measure between $0°$ and $360°$ that is coterminal with the angle of measure $1290°$ in standard position.

Homework 2/4

TB pg. 474-475 #1-21 (e.o.o), 25, 29, 35, 41, 47