Warm Up 1/27

$a^2+b^2=c^2$

1. State the Pythagorean Identity.

$$\boxed{\sin^2 t+\cos^2 t=1}$$

$a^2+b^2=c^2$

$-b^2 \quad -b^2$

$\sqrt{a^2}=\sqrt{c^2-b^2}$

$a=\pm\sqrt{c^2-b^2}$

2. Based on the Pythagorean Identity, what does $\sin t = ?$

$$\boxed{\sin t=\pm\sqrt{1-\cos^2 t}}$$

$\cos^2 t = \boxed{1-\sin^2 t}$

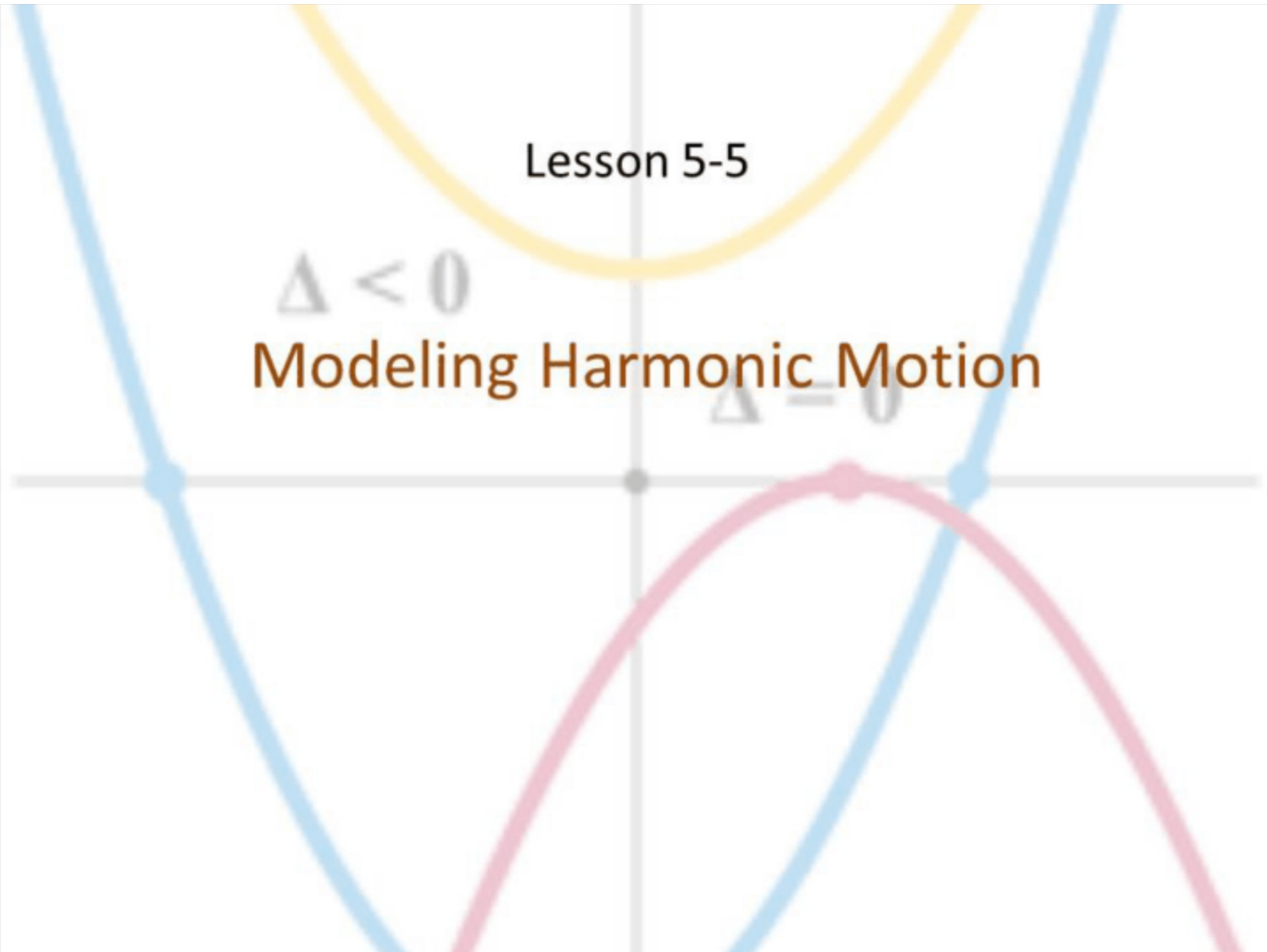
3. Write $\tan^2 t$ in terms of $\sin t$.

$$\tan^2 t=\frac{\sin^2 t}{\cos^2 t}$$

$$=\boxed{\frac{\sin^2 t}{1-\sin^2 t}}$$

Lesson 5-5

Modeling Harmonic Motion



Objective

Students will…

- Be able solve real life problems modeled by sine and cosine functions.
- Be able to understand the physical meaning of amplitude, period, and frequency.

Periodic Behavior

We have learned that sine and cosine functions are **periodic**. It is also clear that many aspects or components of our lives are periodic in nature.

For example, time, along with astronomical movements are periodic (rising/setting of the sun, revolution around the Sun, etc.). Also, soundwaves, microwaves, and other various "waves" are periodic when mapped visually.

Thus, sine and cosine functions have much application to real world situations and problems.

Simple Harmonic Motion

One of the most prominent applications of these periodic functions would be in **harmonic motion**. When we apply sine and cosine functions to harmonic motion, we need to first "re-define" the familiar terms relating to these functions.

$y = a\sin kt$

If the equation describing the displacement y of an object at time t is

$$\boxed{y = a\ sin\omega t} \quad \text{or} \quad \boxed{y = a\ cos\omega t,}$$

then the object is in **simple harmonic motion**. In this case,

amplitude= $|a|$ is the maximum displacement of the object.

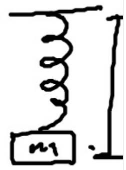
period= $\frac{2\pi}{\omega}$ is the time required to complete one cycle.

frequency = $\frac{\omega}{2\pi}$ is the number of cycles per unit of time.

Note: Treat ω like the way we treated and used k before.

Example

The displacement of a mass suspended by a spring is modeled by the function: $\underline{y = 10\sin 4\pi t}$, where y is measured in inches and t in seconds.



$a = 10$
$\omega = 4\pi$

Find the amplitude, period, and frequency of the motion of the mass.

Amp: $|a| = |10| = 10$ in.

Per: $\dfrac{2\pi}{\omega} = \dfrac{2\cancel{\pi}}{4\cancel{\pi}} = \dfrac{1}{2}$ sec.

Freq.: $\dfrac{\omega}{2\pi} = \dfrac{4\cancel{\pi}}{2\cancel{\pi}} = 2$

Soundwaves

A tuba player plays the note E and sustains the sound for some time. For a pure E the variation in pressure from normal air pressure is given by $V(t) = 0.2 \sin 80\pi t$, where V is measured in pounds per square inch and t in seconds.

a. Find the amplitude, period, and frequency of V.

$a = 0.2$

$\omega = 80\pi$

amp: $|a| = |0.2| = 0.2$

Per: $\frac{2\pi}{\omega} = \frac{2\pi}{80\pi} = \frac{1}{40}$

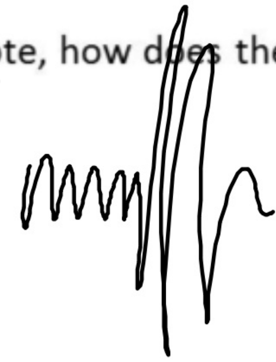
Freq.: 40

a ω

b. If the tuba player increases the loudness of the note, how does the equation for V change? $\boxed{V}(t) = 0.2 \sin 80\pi t$.

a increases

amp increases.



c. If the player is playing the note incorrectly and it is a little flat, how does the equation for V change?

Freq. Changes. ω changed.

A mass is suspended from a spring. The spring is compressed a distance of 4cm and then released. It is observed that the mass returns to the compressed position after $\frac{1}{3}$ second. per: $\frac{1}{3}$ sec.

$a=4$. $y = a\cos\omega t$

a. Find a function that models the displacement of the mass.

$y = \boxed{4\cos 6\pi t}$

$per = \frac{2\pi}{\omega}$

$\frac{1}{3} = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{2\pi}{1/3} = 2\pi \cdot 3 = 6\pi$

b. Find the total displacement after $\overset{=t}{\frac{1}{4}}$ second. (radian)

$\boxed{y = 4\cos 6\pi(1/4) = 0}$

, Find a function that models the simple harmonic motion having the given properties. Assume that the displacement is zero at time $t = 0$.

a) amplitude: 8cm
period: $\frac{1}{4}$ seconds

b) amplitude: 0.8 meters
frequency: $\frac{\pi}{9}$ hertz

Homework 1/27

TB pg. 451 #9-15 (odd), 25(a, c), 31, 33