Warm Up 12/18

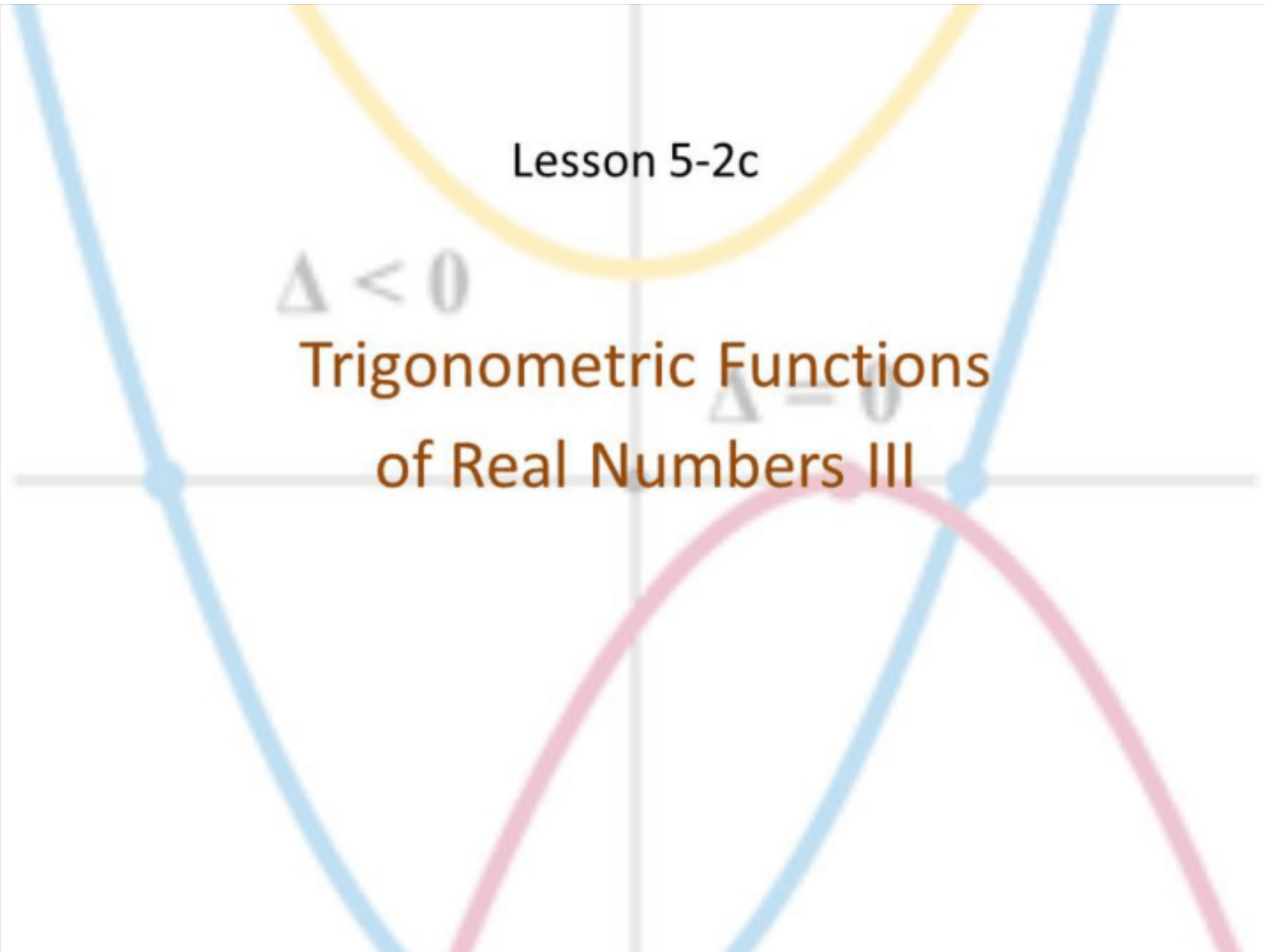
Evaluate the following trig functions without using a calculator.

1. $\sin\frac{2\pi}{3}$

2. $\cos\frac{11\pi}{6}$

3. $\sec\frac{\pi}{4}$

4. $\tan\frac{4\pi}{3}$



Lesson 5-2c

Trigonometric Functions of Real Numbers III

Objective

Students will…

- Be able to rewrite trigonometric functions using the fundamental identities.
- Find all trigonometric functions from the value of one using the fundamental identities.

Trigonometric Functions

Recall from the definitions of trigonometric functions that...

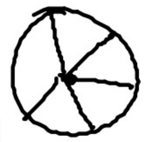
$\tan t = \frac{\sin t}{\cos t} = \frac{Y}{X}$

$\cot t = \frac{1}{\tan t} = \frac{\cos t}{\sin t} = \frac{X}{Y}$

$\sec t = \frac{1}{\cos t}$

$\csc t = \frac{1}{\sin t}$

Coordinates on a Unit Circle



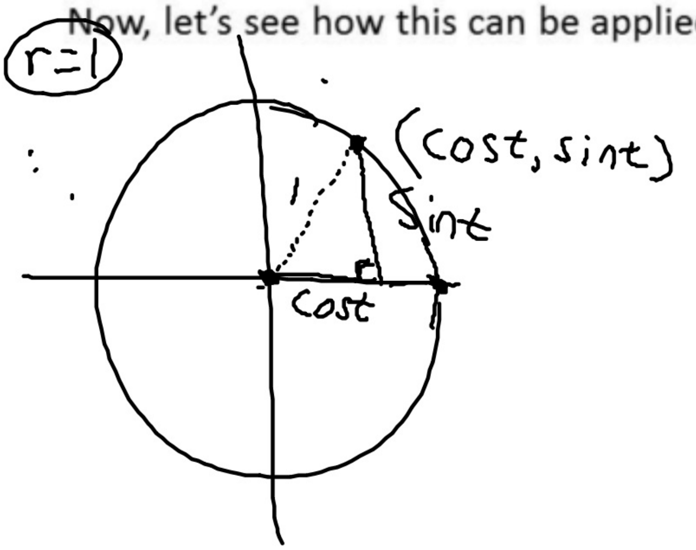
Now, also recall that on the unit circle, we defined the following:

$\cos t = x$ $\sin t = y$ $\rightarrow$ $(x, y) = (\cos t\,, \sin t)$

$(\sin t)^2 = \sin^2 t$

Now, let's see how this can be applied on a unit circle.

$a^2 + b^2 = c^2$



$\sin^2 t + \cos^2 t = 1^2$

$\boxed{\sin^2 t + \cos^2 t = 1}$

Pythagorean Identities

Hence, we can now conclude the following identities:

Pythagorean Identities: (Note: $sin^2 t = (\sin t)^2$)

$sin^2 t + cos^2 t = 1$ $\qquad$ $tan^2 t + 1 = sec^2 t$ $\qquad$ $1 + cot^2 t = csc^2 t$

Also, moving some of these around using algebra:

$\sin t = \pm\sqrt{1 - cos^2 t}$ $\qquad$ $\cos t = \pm\sqrt{1 - sin^2 t}$

$$\begin{aligned} \sin^2 t + \cos^2 t &= 1 \\ -\cos^2 t &= -\cos^2 t \\ \hline \sqrt{\sin^2 t} &= \sqrt{1-\cos^2 t} \end{aligned}$$

$\Rightarrow \sin t = \pm\sqrt{1-\cos^2 t}$

Application of Pythagorean Identities

Although these identities will be used much more extensively in the future, we can still make use of them here.

II | I

III | IV

Example: If $\cos t = \frac{3}{5}$ and t is in quadrant IV, find the values of all other trigonometric functions at t.

$\sin t = -\frac{4}{5}$

We need to first find $\sin t$. We use our identity: $\sin t = \pm\sqrt{1 - cos^2 t}$

$$\sin t = \pm\sqrt{1 - cos^2 t} = \pm\sqrt{1 - \left(\frac{3}{5}\right)^2} = \pm\sqrt{1 - \left(\frac{9}{25}\right)} = \pm\sqrt{\frac{16}{25}} = \pm\frac{4}{5}$$

However, since we are told that t is quadrant IV, $\sin t = -\frac{4}{5}$

(cont.) With this we can now find the rest of the trig. Functions.

$\sin t = -\frac{4}{5}$ $\cos t = \frac{3}{5}$ $\tan t = \frac{\sin t}{\cos t} = \frac{-\frac{4}{5}}{\frac{3}{5}} = \frac{-4}{\cancel{5}} \cdot \frac{\cancel{5}}{3} = \boxed{\frac{-4}{3}}$

$\csc t = \frac{1}{\sin t} = \frac{1}{\frac{-4}{5}} = \boxed{\frac{-5}{4}}$ $\sec t = \frac{5}{3}$

$\cot t = \frac{\cos t}{\sin t}$ $\frac{-3}{4}$

$\frac{1}{\tan t}$

If $\cos t = -\frac{5}{13}$ and t is in quadrant II, find the values of all the trigonometric functions at t.

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$\boxed{\sin t = \frac{12}{13}}$ $\quad \cos t = -\frac{5}{13}$

$\sin t = \pm\sqrt{1-\cos^2 t}$

$= \pm\sqrt{1-\frac{25}{169}}$

$= \pm\sqrt{\frac{144}{169}}$

$= \pm\frac{12}{13}$

$\tan t = \frac{\sin t}{\cos t} = \frac{\frac{12}{13}}{-\frac{5}{13}} = \frac{12}{\cancel{13}} \cdot -\frac{\cancel{13}}{5} = \boxed{-\frac{12}{5}}$

$\cot t = \frac{\cos t}{\sin t} = \frac{1}{\tan t} = \boxed{-\frac{5}{12}}$

$\sec t = \frac{1}{\cos t} = \boxed{-\frac{13}{5}}$

$\csc t = \frac{1}{\sin t} = \frac{13}{12}$

Rewriting Trigonometric Functions

We can also rewrite trigonometric functions using others.

$\cos^2 t = (\cos t)^2$ $\qquad$ $\cos t^2$

Example: Write $\tan t$ in terms of $\cos t$, where t is in quadrant III.

$$\tan t = \frac{\sin t}{\cos t} = \frac{\pm\sqrt{1-\cos^2 t}}{\cos t} = \boxed{\frac{\sqrt{1-\cos^2 t}}{\cos t}}$$

$$= \frac{-y}{-x} = +$$

$3\cdot\sqrt{5}=3\sqrt{5}$

Examples

$\cos t=\pm\sqrt{1-\sin^2 t}$

Write tan t in terms of sin t, where t is in quadrant I

$$\tan t=\frac{\sin t}{\cos t}=\pm\frac{\sin t}{\sqrt{1-\sin^2 t}}\cdot\frac{\sqrt{1-\sin^2 t}}{\sqrt{1-\sin^2 t}}=\boxed{\pm\frac{\sin t\sqrt{1-\sin^2 t}}{1-\sin^2 t}}$$

II	I
III	IV

Write sec t in terms of tan t, where t is in quadrant II

$$\sec t=-\tan^2 t+1$$

$$\sec t=\frac{1}{\cos t}=\frac{1}{x}$$

Homework 12/18

TB pg. 417 #53-61 (odd), 63, 64