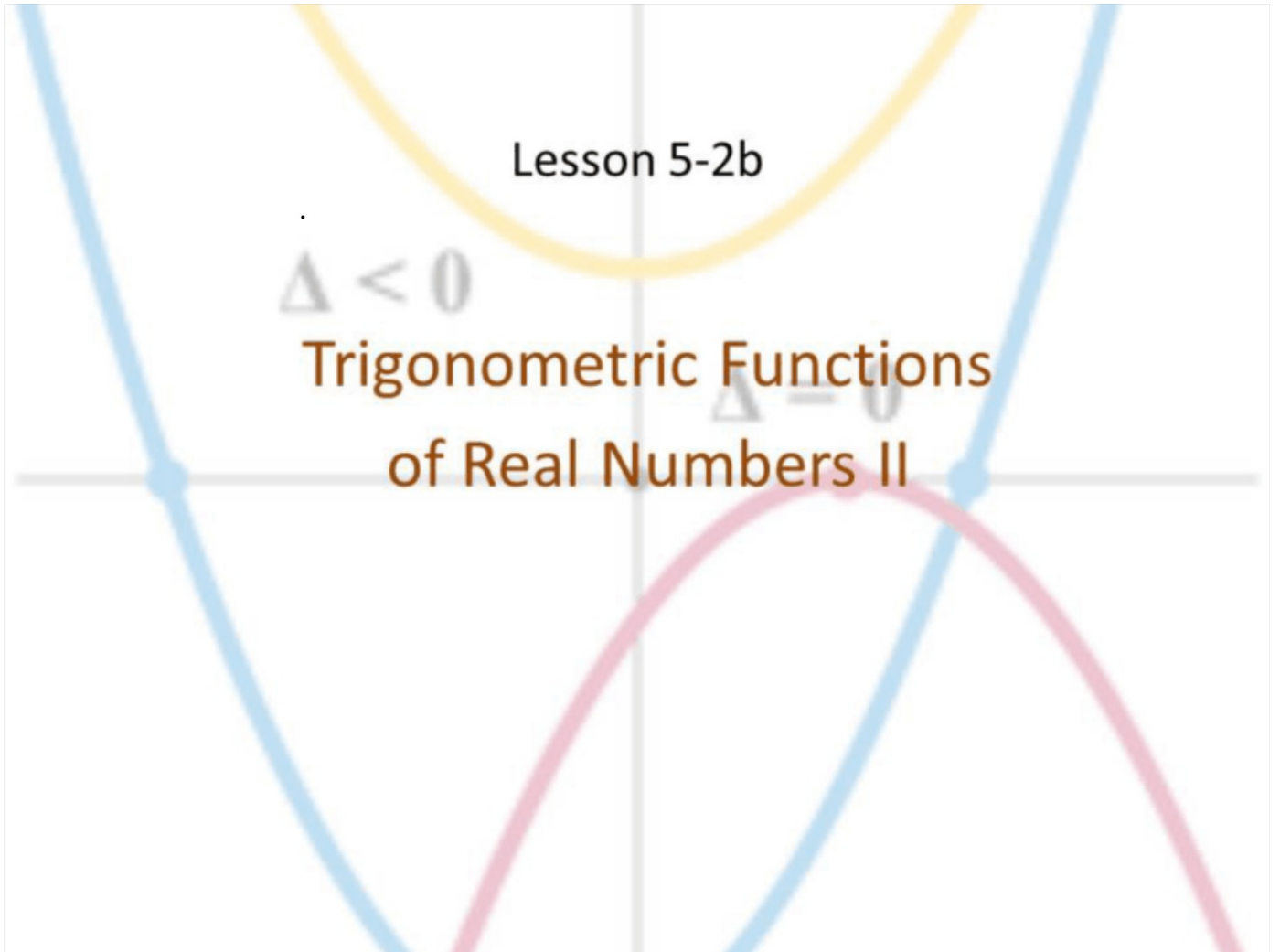


Lesson 5-2b

$\Delta < 0$

Trigonometric Functions
of Real Numbers II

$\Delta = 0$



Objective

Students will...

- Be able to use a calculator to evaluate trigonometric functions (both radians and degree)
- Be able to know and apply the even-odd properties of trigonometric functions.

Trigonometric Functions

The concept of trigonometric functions can be defined in terms of the unit circle. The **definition of trigonometric functions** is as follows:

$$\cos t = x$$

$$\sin t = y$$

$$\tan t = \frac{y}{x} \quad (x \neq 0)$$

$$\sec t = \frac{1}{\cos t} = \frac{1}{x}$$

$(x \neq 0)$

$$\csc t = \frac{1}{\sin t} = \frac{1}{y}$$

$(y \neq 0)$

$$\cot t = \frac{1}{\tan t} = \frac{x}{y}$$

$(y \neq 0)$

Evaluating Trigonometric Functions

The previous definitions of trigonometric functions are only helpful to us if t happens to be one of the values on the unit circle. Consider for example, $\cos \frac{\pi}{7}$

Since $\frac{\pi}{7}$ is not on the unit circle, we would have to use a calculator to evaluate this function.

One thing to keep in mind is that trigonometric functions can be evaluated using both degrees and radians. It is **CRUCIAL** that the calculator is in the right mode. For most calculators, DEG=Degrees, and RAD=Radians.

So, going back to our problem, since $\frac{\pi}{7}$ is a value in radians (no degree sign), we need to put our calculator in radian mode.

$$\cos \frac{\pi}{7} \approx 0.9$$

Try doing these problems. Note that they are all in radians.

$$\sin \frac{\pi}{5} \approx 0.6$$

$$0.59$$

$$\tan \frac{7\pi}{9} \approx -0.83$$

$$\cos \frac{6\pi}{31} \approx 0.82$$

$$\tan \frac{11\pi}{8} \approx 2.41$$

$$\sin \frac{7\pi}{5} \approx -0.95$$

Now, let's try a couple problems in degree mode.

$$\sin 33^\circ \approx 0.54 \quad \cos 67^\circ \approx 0.4 \quad \tan 0.889^\circ \approx 0.015$$

Also, remember that $\sec t = \frac{1}{\cos t}$, $\csc t = \frac{1}{\sin t}$, and $\cot t = \frac{1}{\tan t}$.

~~SIN T~~

$$\text{Thus, } \csc 67^\circ = \frac{1}{\sin 67} \approx 1.09 \quad \text{and} \quad \sec \frac{\pi}{8} = \frac{1}{\cos \pi/8} \approx$$

Try these. Make sure you're in the right mode.

$$\cot \frac{\pi}{19} \approx 5.99 \quad \csc 65.98^\circ \approx \del{0.41} 1.09 \quad \sec \frac{27\pi}{16} \approx 1.8$$

Even-Odd Properties

Consider the following.

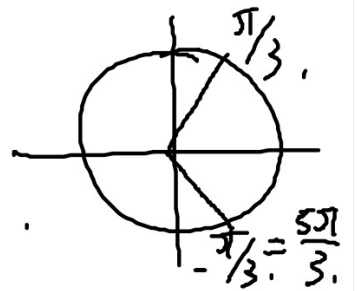
$$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

Now, what about...

$$\cos \frac{\pi}{3} = \frac{1}{2}$$

$$\sin \left(-\frac{\pi}{3}\right) = \sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

$$\cos \left(-\frac{\pi}{3}\right) = \cos\left(\frac{5\pi}{3}\right) = \frac{1}{2}$$



Turns out, these results can be generalized.

Even-Odd Properties:

Even

$$\cos(-t) = \cos t$$

Odd

$$\sin(-t) = -\sin t$$

$$\tan(-t) = -\tan t$$

$$\csc(-t) = -\csc(t)$$

$$\sec(-t) = \sec t$$

$$\cot(-t) = -\cot t$$

$\frac{1}{\sin}$

Examples

$$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

Use the Even-Odd Properties to evaluate the following.

$$\begin{aligned} \sin\left(-\frac{\pi}{6}\right) &= -\left(\sin \frac{\pi}{6}\right) \\ &= -\frac{1}{2} \\ \cot\left(-\frac{5\pi}{6}\right) &= -\cot \frac{5\pi}{6} \\ &= -\frac{1}{\tan \frac{5\pi}{6}} = -\frac{1}{-\frac{1}{\sqrt{3}}} = \sqrt{3} \\ \tan\left(-\frac{11\pi}{6}\right) &= -\tan \frac{11\pi}{6} \end{aligned}$$

$$\begin{aligned} \cos\left(-\frac{\pi}{4}\right) &= \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \\ \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right) \\ \csc\left(-\frac{2\pi}{3}\right) &= -\frac{1}{\sin \frac{2\pi}{3}} = -\frac{1}{\frac{\sqrt{3}}{2}} = -\frac{2}{\sqrt{3}} \\ \sec\left(+\frac{\pi}{2}\right) &= \frac{1}{\cos \frac{\pi}{2}} = \frac{1}{0} \end{aligned}$$

Homework 12/17

Worksheet