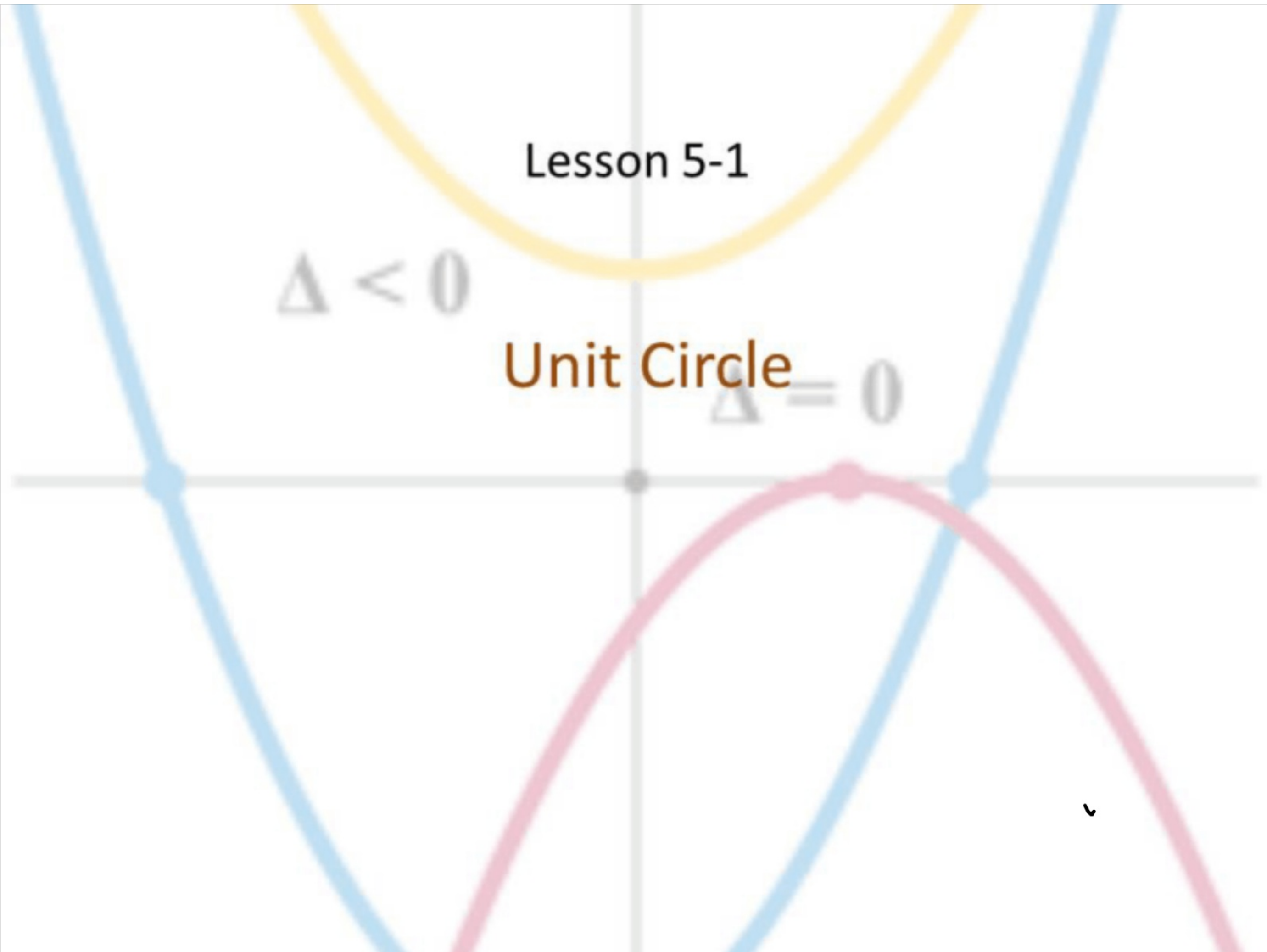

Lesson 5-1
Unit Circle
$\Delta < 0$
$\Delta = 0$

Objective

Students will…

- Be able to complete the Unit Circle

Degree vs Radians

We are generally familiar with the fact that circles measure $360°$. This idea is seen and used often. We can also view it as one revolution around a circle totals $360°$, and halfway around the circle is $180°$ and so on. We can easily fill these into the unit circle.

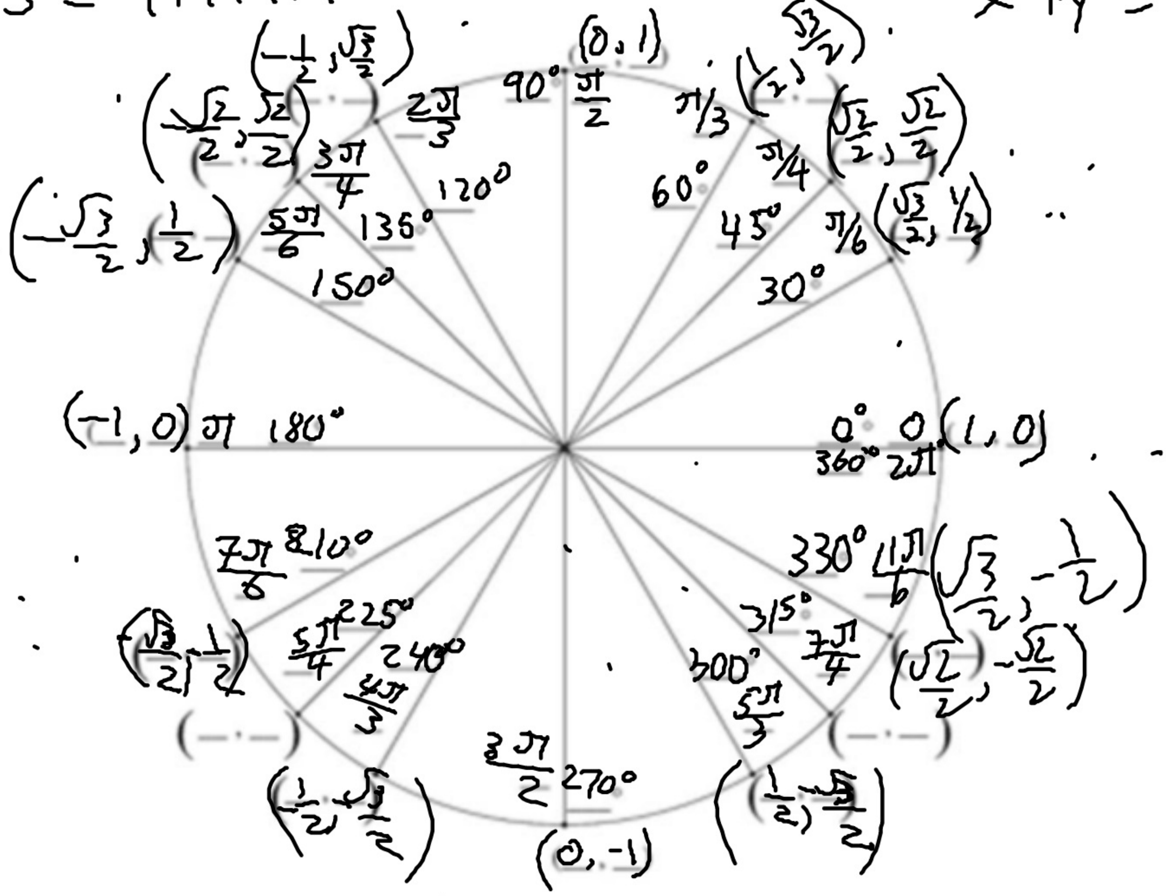
Now, a unit circle can also be measured in a different way. This scale of measure is known as **radians**. The formation of radian measurement arises from trigonometry, but the basic idea of it is that a full revolution around the circle is 2π, which makes π halfway around the circle. We can now fill in the rest. For example, we know that $30° = \frac{1}{3}(90°)$, so if we know that $90° = \frac{\pi}{2}$, then $30° = \frac{1}{3}\left(\frac{\pi}{2}\right) = \frac{\pi}{6}$

$9 \times 5 = 9+9+9+9+9 ..$

$x^2 + y^2 = 1$

$r^2 = 1$

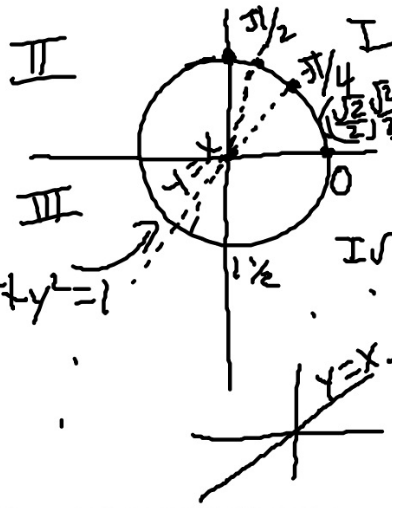
$r = 1$



Coordinates on the Unit Circle

A Unit Circle also has coordinates associated to each radian or degree. These coordinates are derived from the standard equation of the unit circle: $x^2 + y^2 = 1$. Although for the most part this process is rather complicated, there are few coordinates that are simple enough for us to observe. First and foremost, the starting point (0) has coordinates $(1, 0)$. We can fill in the other trivial locations $\left(\frac{\pi}{2}, \pi, \frac{3\pi}{2}\right)$.

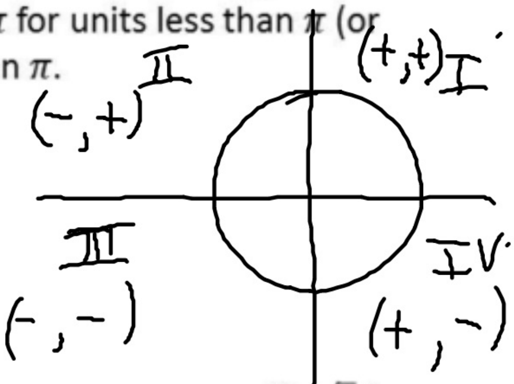
Now, consider the location, $\frac{\pi}{4}$ or $45°$ on the unit circle.



For the others, the calculations are rather tedious and rigorous, so we will simply resort to memorizing. Let us now fill in the coordinates for the first quadrant, 0 to $\frac{\pi}{2}$.

$$\frac{\pi}{6} = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right) \qquad \frac{\pi}{4} = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \qquad \frac{\pi}{3} = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$

Although we must fill in the rest of the circle, we really only need to memorize the first quadrant, and simply use it as a reference for the remaining units. The technique here is to use terminal reference units: π for units less than π (or fraction is less than 1), and 2π for units greater than π.



Example:

$\frac{2\pi}{3}$: Since $\frac{2\pi}{3} < \pi$ or $\left(\frac{2}{3} < 1\right)$, we use π.

So, $\pi - \frac{2\pi}{3} = \frac{\pi}{3}$, which means we use the coordinates $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

The only thing left to do now is to figure out the signs based on which quadrant it is in. We can see that $\frac{2\pi}{3}$ is in the second quadrant, since $\frac{\pi}{2} < \frac{2\pi}{3} < \pi$. Thus, the coordinates are $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

Examples

Let's now try…

$\frac{4\pi}{3}$: Since $\frac{4\pi}{3} > \pi$ (i.e. $\frac{4}{3} > 1$), we need to use 2π.

You try...

$\frac{3\pi}{4}$:

$\frac{11\pi}{6}$:

Homework 12/12

Finish filling in the rest of the Unit Circle.