Warm Up 12/06

Solve.

1) $7^{\frac{x}{2}} = 5^{1-x}$

2) $\log_2(x^2 - x - 2) = 4$

$$x^2 - x - 2 = 16$$
$$-16 \quad -16$$
$$x^2 - x - 18 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{1 \pm \sqrt{1 - 4(1)(-18)}}{2} = \frac{1 \pm \sqrt{73}}{2}$$

Lesson 4-5

Modeling with Exponential Functions

Objective

Students will…

- Be able to solve word problems involving exponential growth/decay.

Exponential Functions in the Real World

Many processes that occur in nature, such as population growth, radioactive decay, heat diffusion ,and numerous others, can be modeled using exponential functions.

We already saw that the exponential number e has a big role when it comes to compounding interest. This number, e, has many other significant roles in real life.

Exponential Growth

One of the most significant situations involving e is the **exponential growth model**.

Exponential Growth Model- A population that experiences exponential growth increases according to the model,

$$n(t) = n_0 e^{rt} \text{ where}$$

$n(t)$ = population at time t

n_0 = initial size of the population

r = relative rate of growth

t = time

Examples

The initial bacterium count in a culture is 500. A biologist later makes a sample count of bacteria in the culture and finds that the relative rate of growth is 40% per hour.

$n(t) = n_0 e^{rt}$

a) Find a function that models the number of bacteria after t hours.

$n(t) = 500e^{0.4t}$

$n_0 = 500$
$r = 0.4$
$t = t$

b) What is the estimated count after 10 hours?

$n(10) = 500e^{0.4(10)} \approx 27{,}299$

In 2000 the population of the world was 6.1 billion and the relative rate of growth was 1.4% per year. It is claimed that a rate of 1.0% per year would make a significant difference in the total population in just a few decades. Test this claim by estimating the population of the world in the year 2050 using a relative rate of growth of a) 1.4% per year and b) 1.0% per year.

$n(t) = n_0 e^{rt}$

a)

$n_0 = 6.1$ billion

$r = 0.014 \quad t = 50$

$n(50) = 6.1e^{(.014 \cdot 50)}$

12.3 billion

b)

$n_0 = 6.1$ billion

$r = 0.01 \quad t = 50$

$n(50) = 6.1e^{(0.01 \cdot 50)}$

10.1 billion

A certain breed of rabbit was introduced onto a small island about 8 years ago. The current rabbit population on the island is estimated to be 4100, with a relative growth rate of 55% per year.

$n(t) = n_0 e^{rt}$

a) What was the initial size of the rabbit population?

$n_0 = ?$

$n(t) = 4100$

$r = 0.55$

$t = 8$

$$\frac{4100}{e^{(0.55 \cdot 8)}} = \frac{n_0 e^{(0.55 \cdot 8)}}{e^{(0.55 \cdot 8)}}$$

$n_0 \approx 50$

b) Approximately, how long will it take for the population to reach 12000?

$n_0 = 50$

$n(t) = 12{,}000$

$r = 0.55$

$t = ?$

$$\frac{12000}{50} = \frac{\cancel{50} e^{(0.55)t}}{\cancel{50}}$$

$240 = e^{0.55t}$

$\ln 240 = \cancel{\ln} e^{0.55t}$

$$\frac{0.55t}{0.55} = \frac{\ln 240}{0.55}$$

$$t = \boxed{\frac{\ln 240}{0.55}} \approx 10$$

Radioactive Decay

Exponential function is also often used to model some form of decay. One of the most significant use is done modeling radioactive decay.

Radioactive Decay- Decay of radioactive mass is modeled by,

$$m(t) = m_0 e^{-rt}$$ where,

$r =$ the rate of decay

$m_0 =$ the initial mass

$h =$ half-life

$t =$ time remaining

The rate r can be then modeled as $r = \frac{ln2}{h}$

Examples

Polonium-210 has a half-life of 140 days. Suppose a sample of this substance has a mass of 300mg.

$$r = \frac{\ln 2}{h} = \frac{\ln 2}{140} \approx 0.004$$

a) Find a function that models the amount of the sample remaining at time t. (Need to find r first)

$t = t$

$m_0 = 300$

$r = 0.005$

$$m(t) = m_0 e^{-rt} \qquad m(t) = 300e^{-0.004t}$$

b) Find the mass remaining after one year.

$t = 365$

$$m(365) = 300e^{(-0.005 \cdot 365)} \approx 48.4\text{ mg}$$

c) How long will it take for the sample to decay to a mass of 200 mg?

$$\frac{200}{300} = \frac{300e^{-0.005t}}{300}$$

$$\frac{2}{3} = e^{-0.005t}$$

$$\ln\left(\frac{2}{3}\right) = \ln\left(e^{-0.005t}\right)$$

$$\frac{\ln\left(\frac{2}{3}\right)}{-0.005} = \frac{-0.005t}{-0.005}$$

$$t \approx 81$$

The half-life of radium-226 is 1600 years. Suppose we have a 22-mg sample.

a) Find a function that models the mass remaining after *t* years.

$r = \frac{\ln 2}{h}$

$m(t) = m_0 e^{-rt} = \boxed{22e^{-0.0004t}}$

$m_0 = 22$

$t = t$

$r = \frac{\ln 2}{1600} \approx 0.0004$

b) How much of that sample will remain after 4000 years?

$m(4000) = 22e^{(-0.0004 \cdot 4000)} \approx 4.44\text{mg}$

c) After how long will only 18 mg of the sample remain?

$\frac{18}{22} = \frac{\cancel{22}e^{-0.0004t}}{\cancel{22}}$

$\frac{9}{11} = e^{-0.0004t}$

$\ln\left(\frac{9}{11}\right) = \cancel{\ln}(e^{-0.0004t})$

$t \approx 502$

Homework 12/06

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