Warm Up 12/03

law of exp

Evaluate the following expression.

$\cancel{a^3 + a^3}$ $\quad$ $c^7 \cdot c^7$

1) a^2a^5 $\quad$ 2) $a^3 + a^3$ $\quad$ 3) $(c^7)^2$

a^7 $\quad$ a^6, a^9 $\quad$ $= c^{14}$

$2a^3$

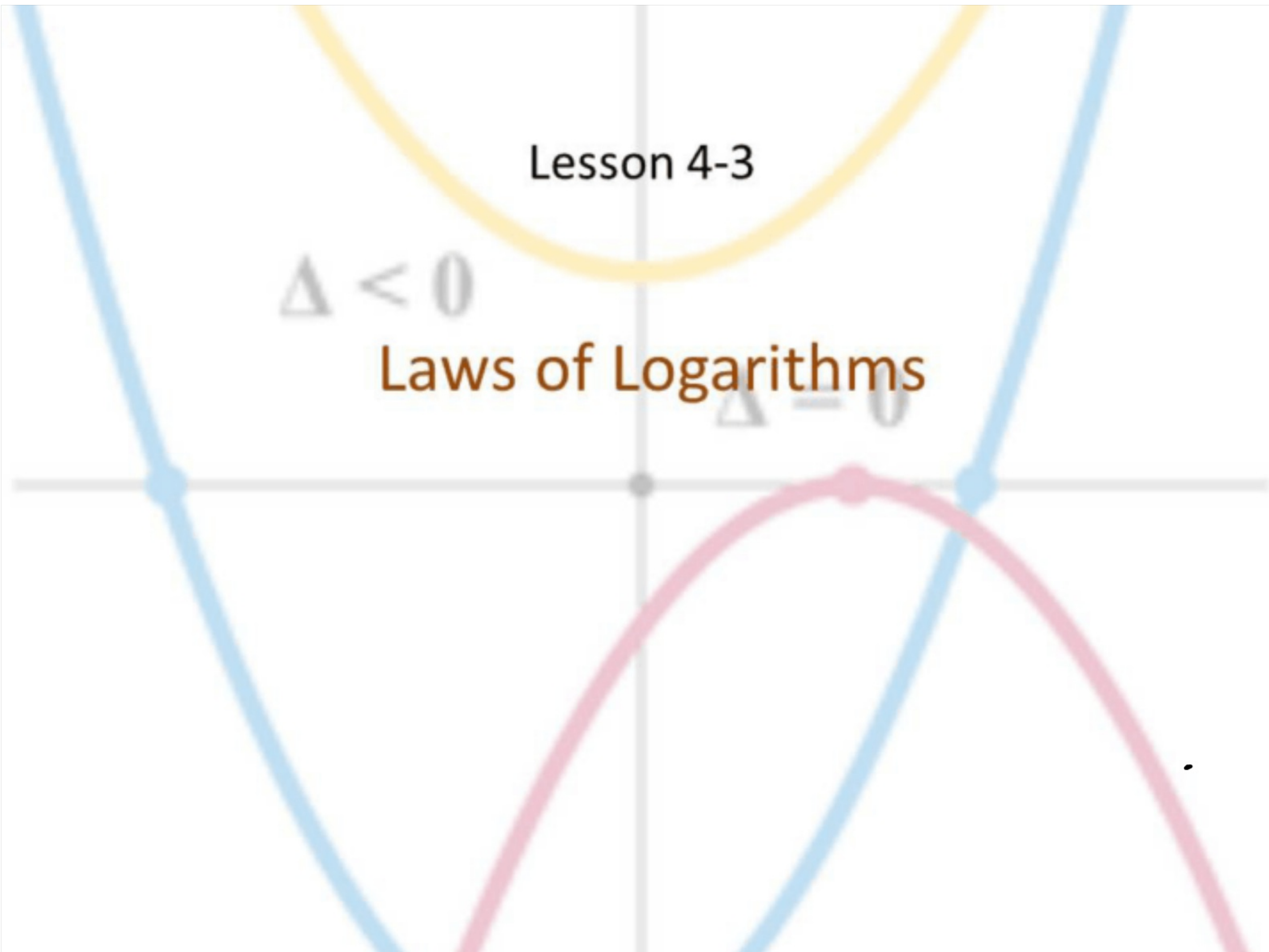
$e^{2-6} = e^{-4}$

4) $\frac{d^6}{d^4}$ $\quad$ 5) $\frac{e^2}{e^6}$ $\quad$ 6) $g^7h^6g^{-3}$ 6

d^2 $\quad$ $\frac{1}{e^4}$ $\quad$ g^4h

Lesson 4-3

Laws of Logarithms



Objective

Students will…

- Be able to know and apply the laws of logarithms.
- Be able to use change of base formula to evaluate logarithms.

Laws of Exponents

$2^2 = 4$.
$\log_2 4 = 2$

When we learned about exponents, we learned that there are certain set of rules regarding exponents, called **The Laws of Exponents**.

For example, when we multiply the same base number with exponents, we simply add the exponents together (i.e. $a^3 a^4 = a^{3+4} = a^7$).

On the other hand when we divide the same base number with exponents, we would subtract the exponents (i.e. $\frac{a^8}{a^7} = a^{8-7} = a^1$).

Lastly, when we take the exponent to an exponent, we would multiply the two exponents together (i.e. $(a^6)^7 = a^{6\times7} = a^{42}$)

Laws of Logarithms

Now, recall that the answers (the output) of logarithms are exponents. For example, we now know that $log_4 x = 6$ is equivalent to $4^6 = x$. Hence, there also exists **Laws of Logarithms**, much similar to the Laws of Exponents.

$1^x = 1$

Laws of Logarithms- Let a be a positive number, with $a \neq 1$. Let A, B, and C be any real numbers with $A > 0$ and $B > 0$.

Laws

1. $log_a(AB) = log_a A + log_a B$
2. $log_a\left(\frac{A}{B}\right) = log_a A - log_a B$
3. $log_a(A^c) = Clog_a A$

Just like the Laws of Exponents, it is **imperative** to have the **same** base!

Examples

By Law 1,

$A \quad B \qquad\qquad\qquad\qquad\qquad\qquad\qquad \log_a A + \log_a B$

$$\log_2(8 * 32) = \log_2(256) = \underline{8} = (3 + 5) = \log_2(8) + \log_2(32)$$

$a \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 3 \qquad 5$

By law 2,

$$\log_2\left(\frac{32}{8}\right) = \log_2(4) = \underline{2} = (5 - 3) = \log_2(32) - \log_2(8)$$

$5 - 3$

By law 3,

$$\log_2(4)^2 = \log_2 16 = 4 = 2(2) = 2\log_2(4)$$

Examples

Evaluate the following expressions.

a) $\underbrace{log_4 2}_{}$ + $log_4 32$ $\quad \frac{1}{2}$

$log_4(32 \times 2)$

$= log_4(64) = 3$

b) $log_6 4 + log_6 9$

$log_6(4 \cdot 9) = log_6(36)$

$= 2$

c) $log_5 75 - log_5 3$

$log_5\left(\frac{75}{3}\right)$

$= log_5(25)$

$= 2$

d) $log_2 80 - log_2 5$

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Examples

Use the Laws of Logarithms to expand each expression.

a) $log_2(6x)$

$\boxed{\log_2 6 + \log_2 x}$

b) $log_5(x^3y^6)$

$\log_5 x^3 + \log_5 y^6$

$= 3\log_5 x + 6\log_5 y$

$3(\log_5 x + 2\log_5 y)$

c) $\ln\left(\frac{ab}{\sqrt[3]{c}}\right)$

$\log_e$

$c^{1/3}$

$= \ln(c^{1/3})$

$\ln(ab) - \ln(\sqrt[3]{c})$

$\ln(a) + \ln(b) - \frac{1}{3}\ln(c)$

Change of Base

The last thing we need to cover in this section is the change of base formula. There were two "special" bases for logarithms: base 10 and e. Thus, for computing logarithms using a calculator, our goal is to **change the base** of our expression to either base 10 or e. This can be done by:

$\log_2 4$ $\log_2 5$

Change of Base Formula:

$$log_b x = \frac{log_a x}{log_a b}$$

This formula allows us to change the base of our logarithm to any base we want to. However, as mentioned, we want to change the base to either base 10 or e. Be aware that base e is used more.

Examples

Use the change of base formula and calculator to evaluate.

a) $log_5 8$

$$\frac{\log 8}{\log 5}$$

$= 1.29$

b) $log_9 20$

$$\frac{\ln(20)}{\ln(9)}$$

$= 1.36$

In Closing

Expand or combine the following using the laws of logarithms and check your answers with a partner.

1) $log_4 \frac{x}{2}$

2) $\log 12 + \frac{1}{2}\log 7 - \log 2$

Homework 12/03

TB pg. 356-357 #1, 11, 12, 13, 27, 39, 44, 49, 52, 53