# Warm Up 12/02

Identify the base of each log or exponential function.

$9^x = 625$

1) $2^x = 8$ 2) $log_4 x = 2$ 3) $3^x = 27$

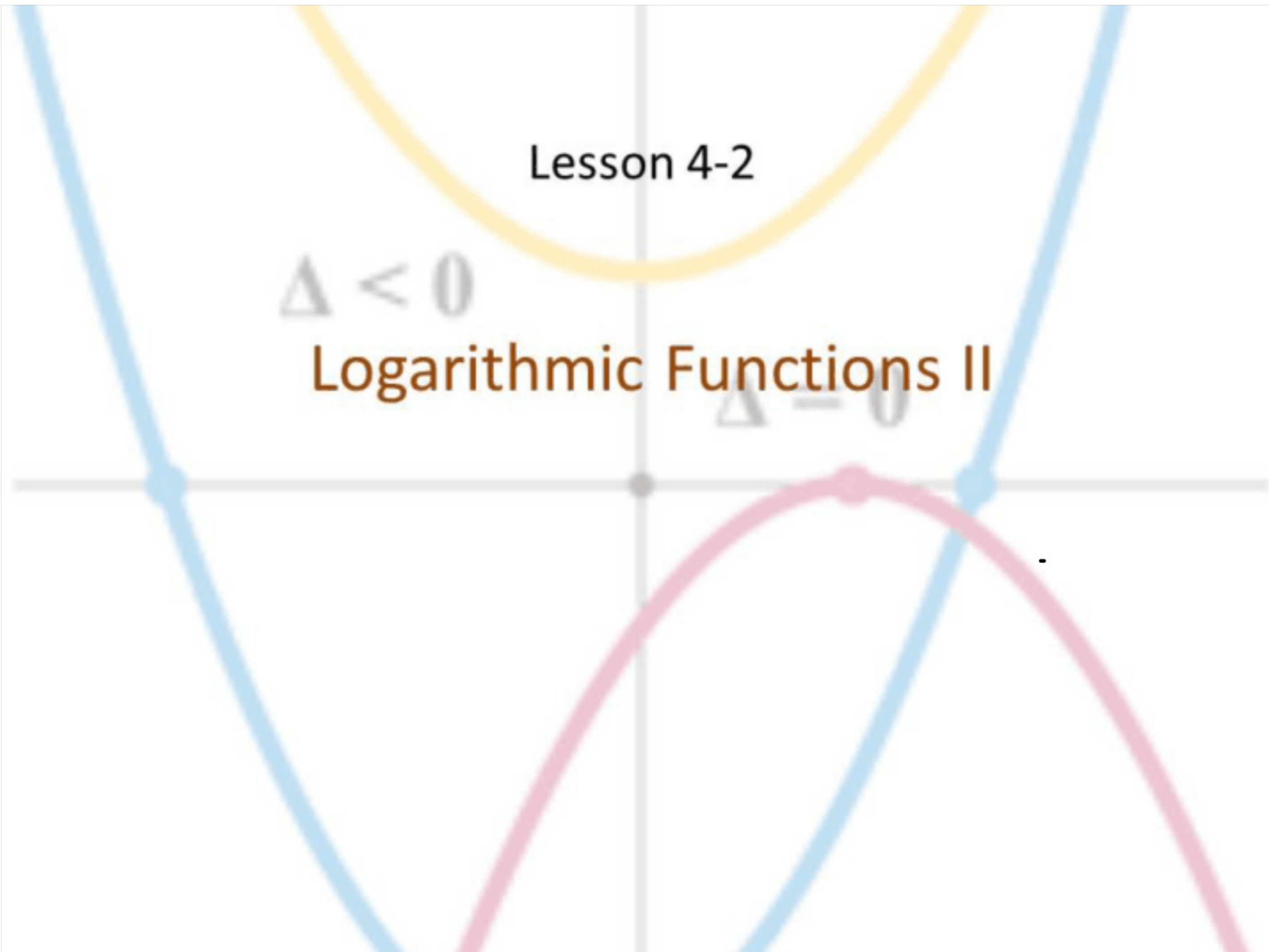
2 4 3

4) $log_9 x = 625$ 5) $e^x = 1$ 6) $\log x = 4$

9 e 10

Lesson 4-2

# Logarithmic Functions II



# Objective

Students will…

- Be able to define natural logarithmic function.
- Be able to know and apply the properties of natural logarithms.
- Be able to use calculators to compute natural logarithms.

# Natural Logarithms

We've learned that any logarithm with base 10 is known as the *common* logarithm, without the base written. In our previous section of exponential function, we learned about a very special number denoted $e$. Naturally (no pun intended as we'll see), logarithms with base $e$ is also considered special, and it is given a special name.

log

**Natural Logarithm**- The logarithm with base $e$ is called the **natural logarithm** and is denoted by **ln**:

$$\ln x = log_e x$$

# The Inverse of Exponential Function

Like all other exponential and logarithmic functions, the natural logarithmic function $y = \ln x$ is the inverse function of the exponential function $y = e^x$. Hence, by definition we have

$$\ln x = y \leftrightarrow e^y = x$$

$\log_e x = y$ $\quad e^y = x$

Example:

"=" 

$$e^6 \approx 403.43 \quad \rightarrow \quad \ln 403.43 \approx 6$$

$$\ln 8 \approx 2.08 \quad \rightarrow \quad e^{2.08} \approx 8$$

# Properties of Natural Logarithms

We have learned about some of the basic properties of logarithms. Always remember that, although it's given a special name, natural logarithms is still a logarithmic function! Thus, the properties of natural logarithm naturally (again, no pun intended ☺) follow the properties of logarithms. Simply replace $a$ with $e$ and $log_a$ with $ln$.

| Property | Reason |
|---|---|
| 1. $\ln 1 = 0$ $e^0 = 1$ | Anything raised to the zero power is 1 |
| 2. $\cancel{\ln e} = 1$ $\cancel{\log_3 3}$ <br> $\cancel{\log_e e}$ | Anything raised to the 1st power is itself |
| 3. $\cancel{\ln e}^{x} = x$ | $e$ raised to the $x$ power is $e^x$ |
| 4. $\cancel{e}^{\ln x} = x$ | $\ln x$ is the power to which $e$ must be raised to get $x$ |

# Examples

For base $e$...

By property 1:

$$\ln 1 = 0$$

By property 2:

$$\cancel{\ln e} = 1$$

By property 3:

$$\cancel{\ln e}^{8} = 8$$

By property 4:

$$\cancel{e}^{\ln 12} = 12$$

$2^x$

$e^{(x)}$

$\cancel{\ln(e)}$

$\cancel{e}^{\ln(\ldots)}$

$e \times \ln(\cdot)\,\cancel{e}^{\ln(\ldots)}$

# You try

By property 1:

$$\ln 1 = 0$$

By property 2:

$$\cancel{\ln e} = 1$$

By property 3:

$$\cancel{\ln e}^4 = 4$$

By property 4:

$$\cancel{e^{\ln}}^{19} = 19$$

# Using a Calculator

For most logarithmic, as well as exponential functions, we've learned that having a calculator is a must. Computing natural logarithm on a calculator is easy. We simply need to find where the $ln$ button is. Almost all calculators place $e^x$ and $ln$ together [redacted]

Example:

"ln"

To compute $\ln 5$, we would input "[redacted]", then "5".

The answer should read: $\ln 5 = 1.6094379124341$

1.61

1.60

# In Closing

Compute the following natural logarithms using a calculator and check your answers with a partner.

1) $\ln 4 =$ 1.39

2) $(2)(\ln 9) =$ 4.39

3) $9(\ln 11) =$ 21.6

Homework 12/02

TB pg. 349-350 #7, 8, 13, 14, 22a, 23b, 23c, 35, 36

$\log_e$

$\ln y = 5 \qquad \log_2 y = 5$

$e^5 = y \qquad 2^5 = y$