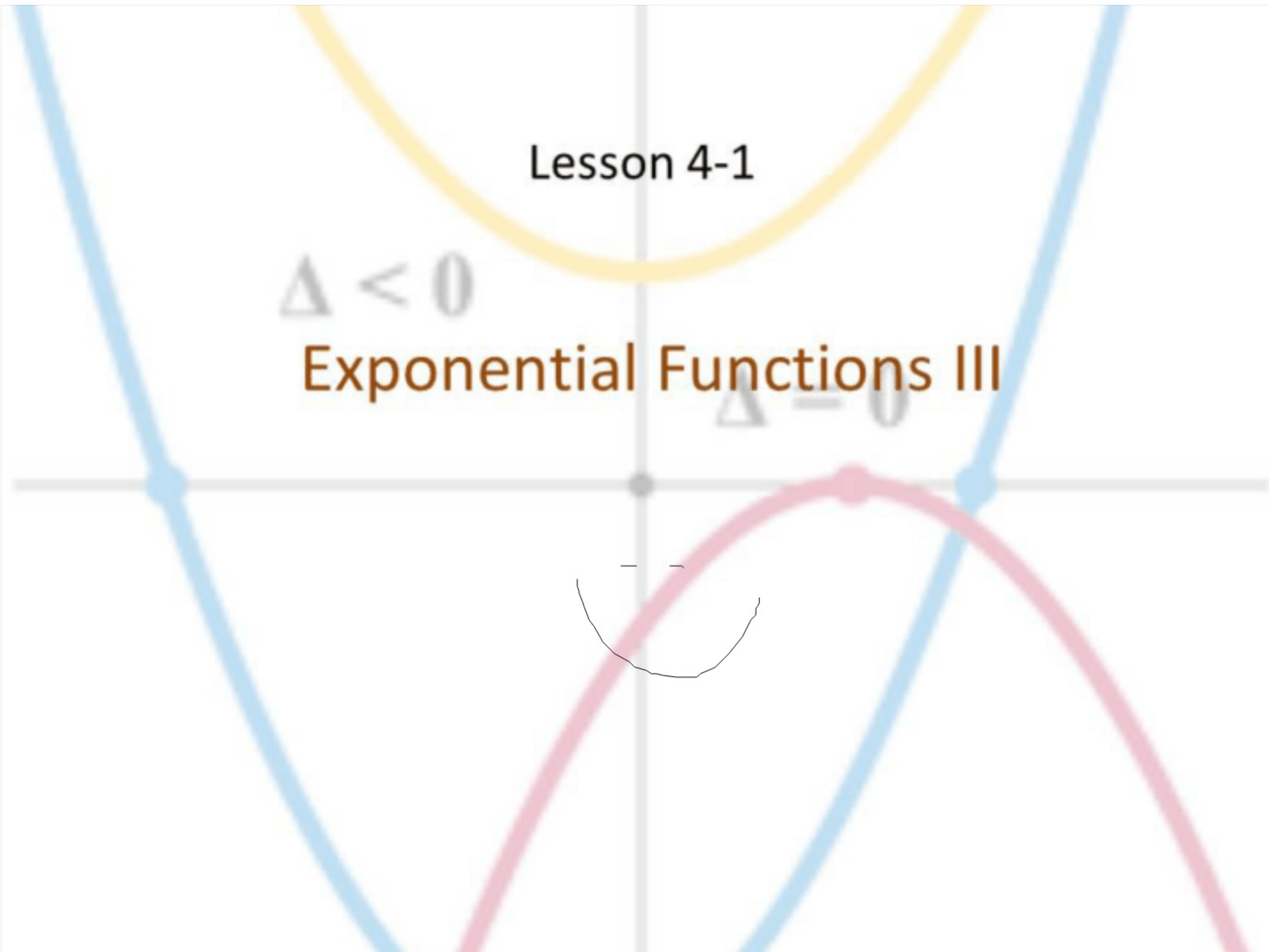
Lesson 4-1

Exponential Functions III



Objective

Students will…

- Be able to derive the exponential function from a given graph
- Be able to recognize the base e.
- Be able to solve interest problems using the natural exponential function, $f(x) = e^x$

Exponential Functions

In our previous chapter, we studied polynomial and rational functions. Yet another important and practical function group is the exponential function.

The **<u>exponential function</u>** with **base** a is defined for all real numbers by

$f(x) = a^x$, where $a > 0$ and $a \neq 1$.

Deriving Exponential Functions

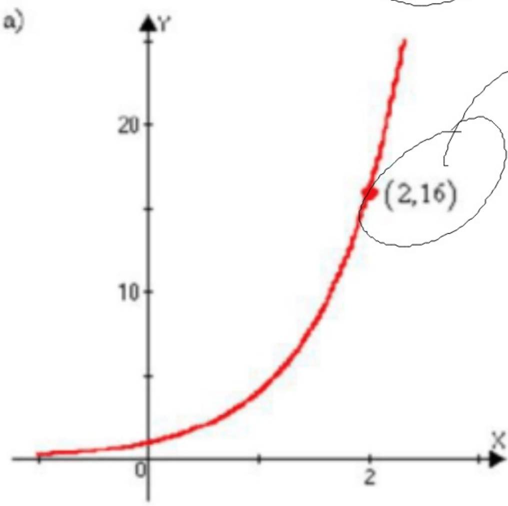
With that said, we can also find the equation of the functions from the given graphs. The idea is to use the exponential definition,

$$f(x) = a^{x}$$

in.

out

Ex. a)



(2,16)

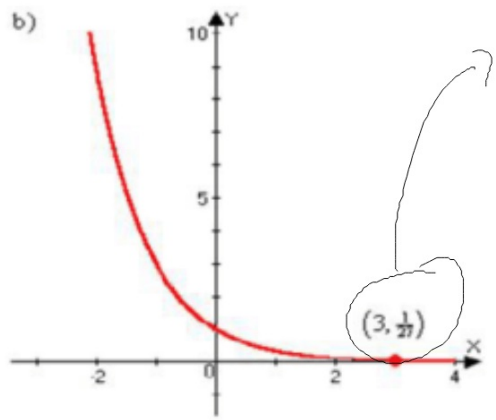
$\sqrt{16} = \sqrt{a^2}$

$\pm 4 = a$

$\boxed{f(x) = 4^x}$

Examples

Find the exponential function $f(x) = a^x$ whose graph is given.

1. 

$(3, \frac{1}{27})$

$$\sqrt[3]{\frac{1}{27}} = \sqrt[3]{a^3}$$

$$a = \frac{1}{3}$$

$$f(x) = \left(\frac{1}{3}\right)^x$$

The *Natural* Exponential Functions

In studying exponential functions, there is a very special number that is studied, mainly because of its use virtually on a daily basis out in the real world. It is ~~a~~ called the *Natural* exponential function, denoted as e

So, by definition, the ***natural* exponential function** is the exponential function

$$f(x) = e^x\text{, where the base } e \approx 2.71828\ \ldots$$

By definition e is the value that $\left(1 + \frac{1}{n}\right)^n$ approaches as $n \to \infty$

This will be studied much more extensively in Calculus. For this course, our focus is simply using this strange number via a calculator ☺

Compound Interest

The question may be why use such a strange and random base? Actually, it turns out that this little e has much use out in the real world. Again, in Calculus you will see that e isn't all that "random," and have a better idea why e has so much use out in the real world. Here's an example: Calculating Compound Interest! Ka-ching!

Compound Interest is calculated by the formula: $A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$,

where $A(t) =$ the amount after t years

$P =$ the Principal amount (initial amount put in)

$r =$ the interest rate per year

$n =$ the number of times interest~~ed~~ is compounded per year

$t =$ the number of years

Example

A sum of $1000 is invested at interest rate of 12% per year. Find the amounts in the account after 3 years if interest is compounded annually, semiannually, quarterly, monthly, and daily.

Here we need to use our compound interest formula.

$$A(t) = P\left(1+\frac{r}{n}\right)^{nt}$$

Annually: $1000\left(1+\frac{0.12}{1}\right)^{(1)(3)}$

$= 1404.93$

Daily: $1000\left(1+\frac{0.12}{365}\right)^{(365)(3)}$

$= 1433.24$

Continuously Compounded Interest

For some accounts, the interest is compounded continuously, rather than periodically (like our previous problem). For this kind of compounding, the formula is actually much simpler.

<u>Continuously Compounded Interest</u> is calculated by,

$$A(t) = Pe^{rt} \quad \text{where,}$$

$A(t) =$ the amount after t years

$P =$ the Principal amount (initial amount put in)

$r =$ the interest rate per year

$t =$ the number of years

Example

Find the amount after 3 years if \$1000 is invested at an interest rate of 12% per year, compounded continuously.

$A(t) = Pe^{rt}$

$(1000)e^{(.12)(3)} = \$1433.33$

Homework 11/15

TB pg. 336, 340 #15-18, 77(a, b, c, e, g), 81