

## Warm Up 11/13

Let  $f(x) = 5^x$ . Evaluate the following.

1.  $f(4)$

$$5^4 = 625$$

2.  $f(-2^4)$

$$f(-16)$$
$$5^{-16} = \frac{1}{5^{16}}$$

3.  $[f(-2)]^3$

$$\left(\frac{1}{5^2}\right)^3 = \left(\frac{1}{25}\right)^3 = \frac{1}{15,625}$$

4.  $f\left(\frac{3}{2}\right)$

$$5^{(3/2)} = 11.18$$

5.  $f(-\sqrt{3})$

$$5^{-\sqrt{3}} = 0.062$$

6.  $f(2\pi)$

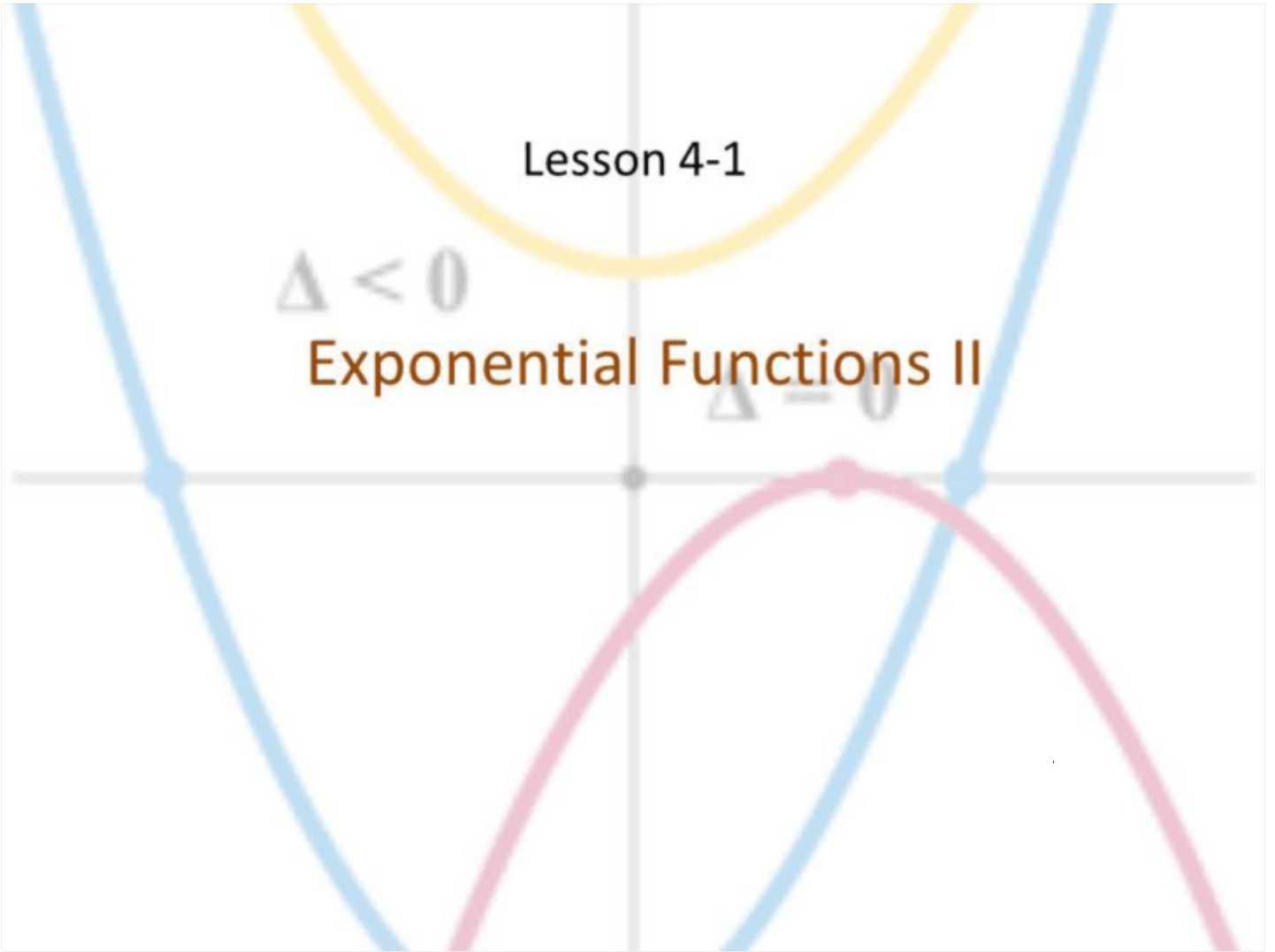
$$24646.7$$

Lesson 4-1

$\Delta < 0$

Exponential Functions II

$\Delta = 0$



## Objective

Students will...

- Be able to identify the end behavior of exponential functions.
- Be able to derive the exponential function of whose graph is given.

## Exponential Functions

In our previous chapter, we studied polynomial and rational functions. Yet another important and practical function group is the exponential function.

The exponential function with **base**  $a$  is defined for all real numbers by

$$f(x) = a^x, \text{ where } a > 0 \text{ and } a \neq 1.$$

Also, note that here our **exponent** is the variable, instead of the **base**.

## The End Behavior of Exponential Functions

During our previous lesson we made a table of values and graphed the two functions,  $f(x) = 3^x$  and  $f(x) = \left(\frac{1}{3}\right)^x$ . Let's try to describe the end behavior of each.

$$f(x) = 3^x$$

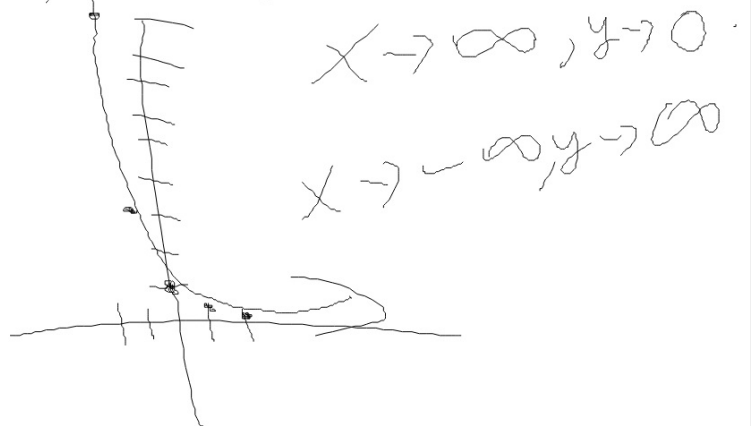
$$\begin{aligned} 3^4 &= \frac{1}{3} \\ 3^2 &= \frac{1}{9} \\ 3^{-3} &= \frac{1}{27} \end{aligned}$$



$$x \rightarrow \infty, y \rightarrow \infty$$

$$x \rightarrow -\infty, y \rightarrow 0$$

$$f(x) = \left(\frac{1}{3}\right)^x$$



$$x \rightarrow \infty, y \rightarrow 0$$

$$x \rightarrow -\infty, y \rightarrow \infty$$

## The "Semi" End Behavior of Exponential Functions

So let's now generalize the "semi" end behavior of exponential functions. For the sake of better studying the end behavior, we are going to consider a slightly modified version of the exponential function.

For  $f(x) = a^x + k$ , where  $k$  is any real number,

$$f(x) = 3^x + 0$$

If  $a > 1$ , as  $x \rightarrow \infty$ ,  $y \rightarrow \infty$ , and as  $x \rightarrow -\infty$ ,  $y \rightarrow k$

If  $a < 1$ , as  $x \rightarrow \infty$ ,  $y \rightarrow k$ , and as  $x \rightarrow -\infty$ ,  $y \rightarrow \infty$

$$f(x) = \left(\frac{1}{3}\right)^x$$

## Examples

Identify the end behavior of the following.

$a > 1$   $k = 0$   
a.  $f(x) = 4^x$

$x \rightarrow \infty, y \rightarrow \infty$

$x \rightarrow -\infty, y \rightarrow 0$

$a > 1$   $k = -8$   
d.  $f(x) = \left(\frac{5}{4}\right)^x - 8$

$x \rightarrow \infty, y \rightarrow \infty$

$x \rightarrow -\infty, y \rightarrow -8$

$a > 1$   $k = 6$   
b.  $f(x) = 4^x + 6$

$x \rightarrow \infty, y \rightarrow \infty$

$x \rightarrow -\infty, y \rightarrow 6$

$a < 1$   $k = -2$   
c.  $f(x) = \left(\frac{1}{4}\right)^x - 2$

$x \rightarrow \infty, y \rightarrow -2$

$x \rightarrow -\infty, y \rightarrow \infty$

$a > 1$   $k = 1$   
e.  $f(x) = 1.2^x + 1$

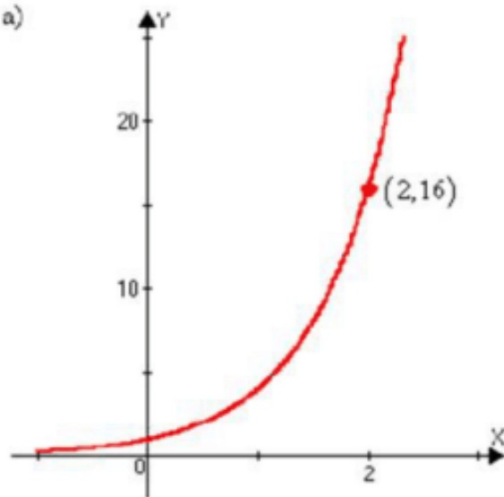
$x \rightarrow \infty, y \rightarrow \infty$

$x \rightarrow -\infty, y \rightarrow 1$

## Deriving Exponential Functions

We can also find the equation of the functions from the given graphs.  
The idea is to use the exponential definition,  $f(x) = a^x + k$

Ex. a)

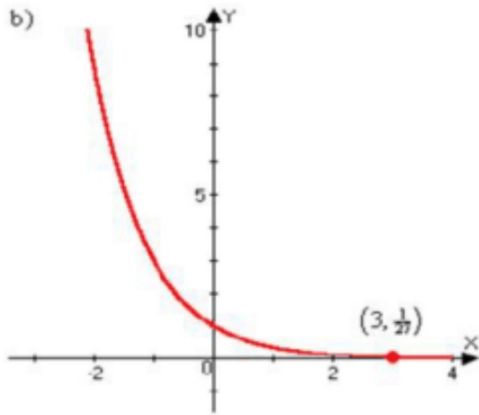




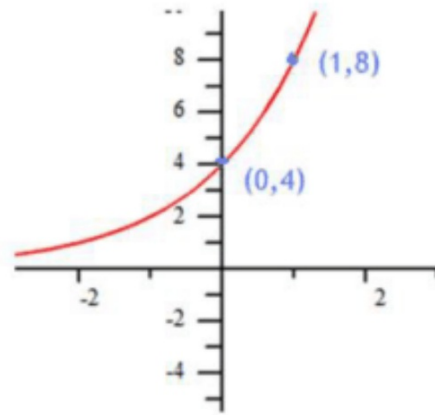
## Examples

Find the exponential function  $f(x) = a^x + k$  whose graph is given.

1. b)



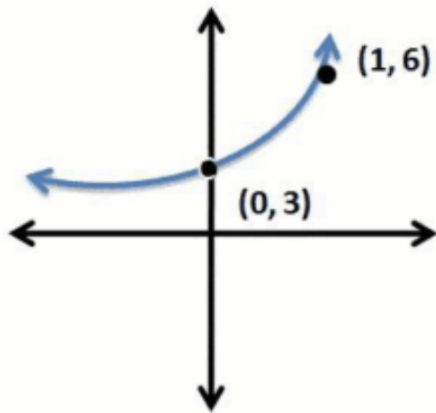
2.



## Examples

Find the exponential function  $f(x) = a^x + k$  whose graph is given.

1.



## Homework 11/13

TB pg. 336-337 #15-18, for 27, 29, 30  
only describe the end behavior, 39, 40