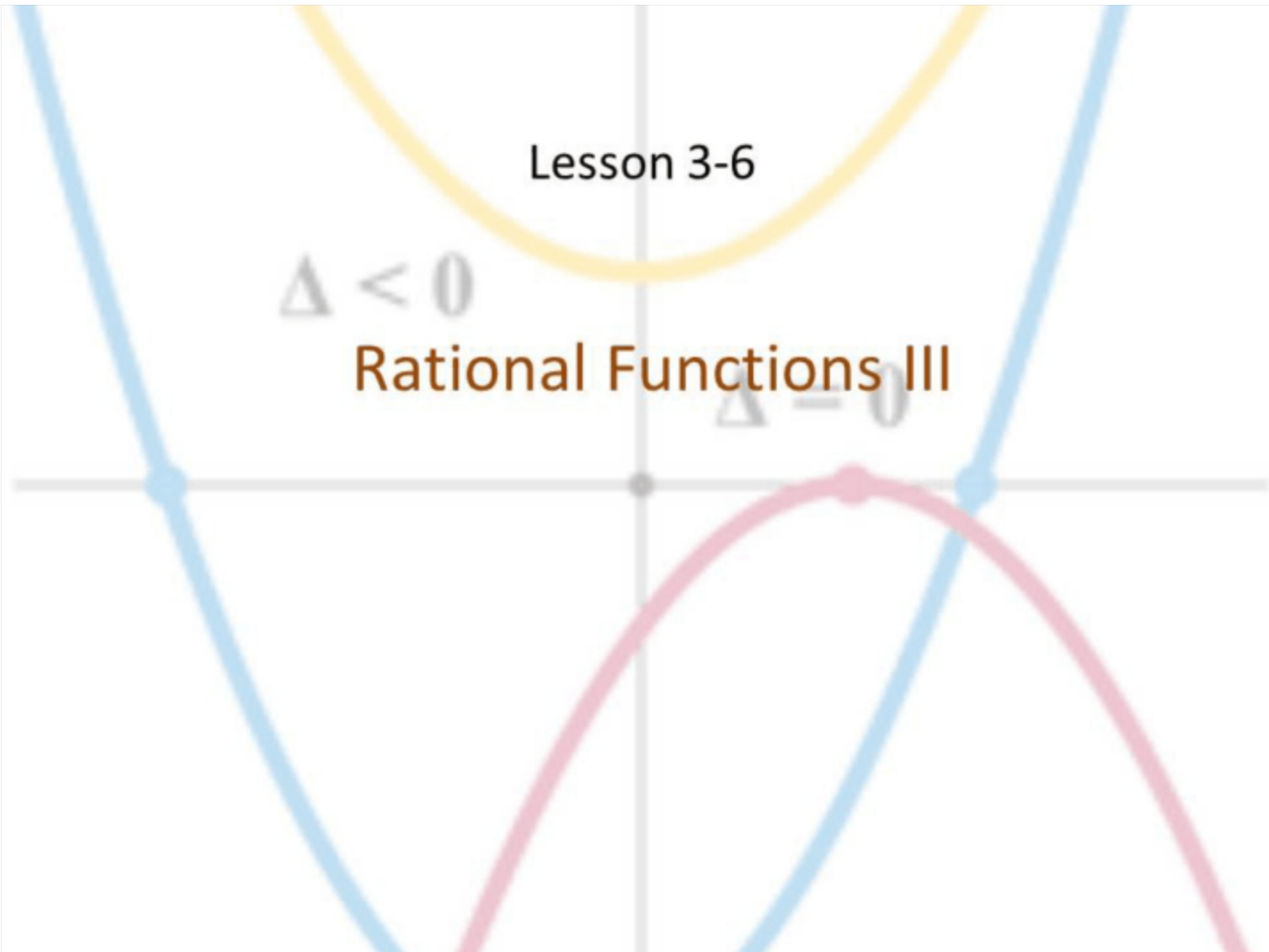

Lesson 3-6
Rational Functions III
Δ < 0

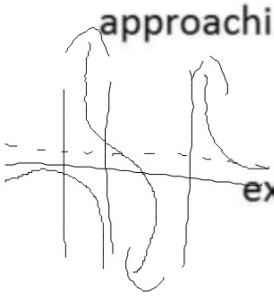
Objective

Students will…

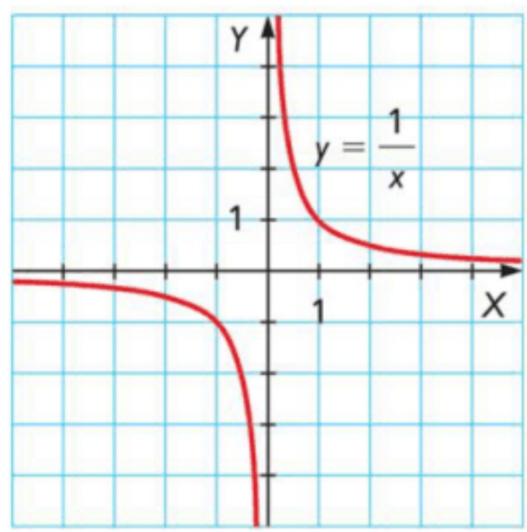
- Be able to identify the behaviors near the asymptotes.

Asymptotes

One of the characteristics of rational function graphs is the presence of asymptotes. **<u>Asymptotes</u>** are lines that the graph of the function gets closer and closer to as it travels on. You may think of them as boundary lines that the graph will never touch, while continually approaching them.



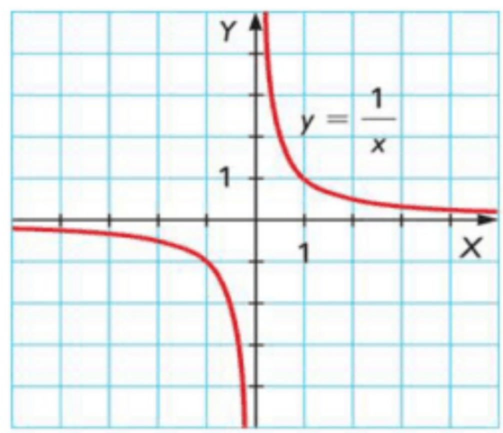
ex. $f(x) = \frac{1}{x}$



We can see that both x and the y-axis are asymptotes of this graph.

Behavior Near Vertical Asymptotes

From the previous graph, we saw that the vertical asymptote for $f(x) = \frac{1}{x}$ is $x = 0$, i.e. the y-axis.



We can observe how the graph behaves near this vertical asymptote.

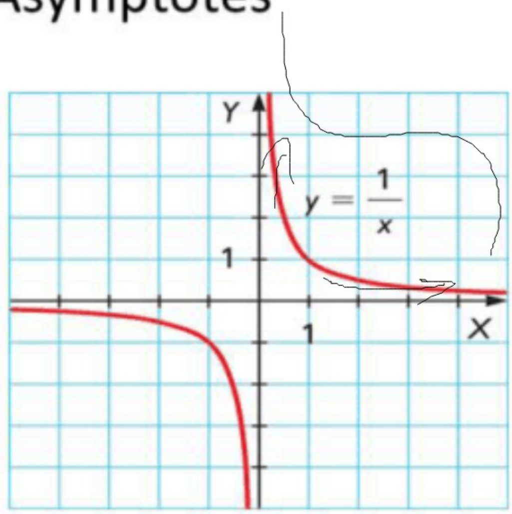
We need to observe the behavior for when the graph approaches $x = 0$ from the right side, as well as from the left side. Such behavior is denoted as follows:

$x \to 0^+$ denotes when, "x approaches zero from the right."

$x \to 0^-$ denotes when , "x approaches zero from the left."

Behavior Near Vertical Asymptotes

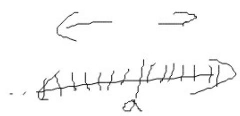
So, with that said, looking at this graph, we can observe that as the graph approaches zero from the right, it goes up, and the graph goes down as it approaches zero from the left.



In other words,

$$x \to 0^{+}, y \to \infty$$

$$x \to 0^{-}, y \to -\infty$$



Behavior Near Vertical Asymptotes

So the behavior near the asymptotes can be easily viewed when looking at a graph, but the graph is not always given. However, we can still find the graph's behavior near the asymptotes quite easily by doing the following:

Let $\mathrm{x} = a$ be the vertical asymptote. Then,

To find the behavior for as $x \to a^{+}$, we need to plug in a number that is to the "right" of *a* (i.e. $a + 1$). $+ ; y \to \infty \ , - : y \to -\infty$

To find the behavior for as $x \to a^{-}$, we need to plug in a number that is to the "left" of *a* (i.e. $a - 1$). $+ : y \to \infty \ , - ; y \to -\infty$

<u>Note:</u> We only need to understand the behavior near the vertical asymptotes.

Examples

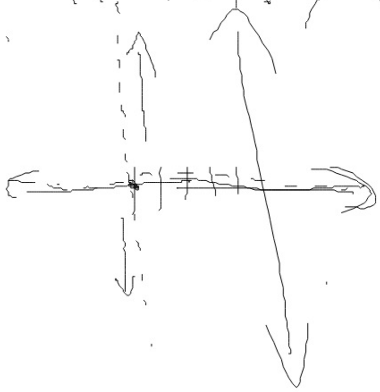
Find the asymptotes of the following functions, and identify the graph's behavior near them.

1. $f(x) = \dfrac{5x+21}{x^2+10x+25}$

V-ASY: $x^2+10x+25=$

$(x+5)(x+5)$

H-ASY: $y=0$



2. $f(x) = \dfrac{x^3+3x^2}{x^2-4}$

V-ASY = $x^2-4=0$, $x=\pm 2$

H-ASY = NONE

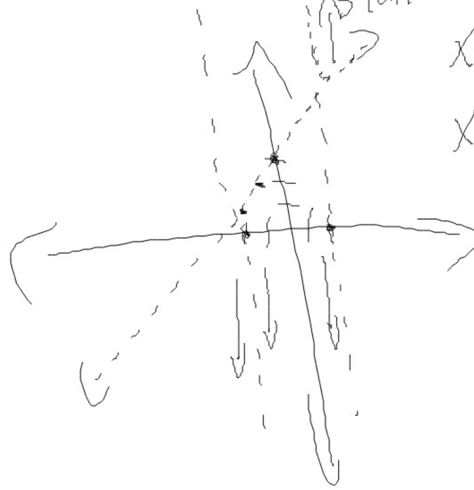
Slant ASY = $y=x+3$

$x \to 2^+,\ y \to \infty$

$x \to 2^-,\ y \to -\infty$

$x \to -2^+,\ y \to -\infty$

$x \to -2^-,\ y \to -\infty$



Examples

3. $f(x) = \frac{x^2-3x-4}{2x^2+4x}$

4. $f(x) = \frac{2x(x+2)}{(x-1)(x-4)}$

Homework 11/5

TB pg. 313 #33-53 (e.o.o)

Just find the x, y-intercepts, asymptotes, and behavior near the asymptotes. Do NOT need to graph!