PreCalculus Name: Period: Date:

10/30

Lesson 3-6b: Rational Functions

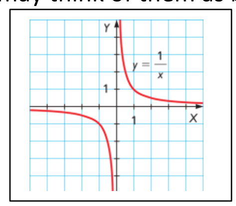
Objective

Students will…

- Be able to identify and solve for both vertical and horizontal asymptotes of rational functions.
- Be able to find the slant or oblique asymptote of a rational function, given that it exists.

Asymptotes

One of the characteristics of rational function graphs is the presence of asymptotes. ____________________ are lines that the graph of the function gets closer and closer to as it travels on. You may think of them as boundary lines that the graph continually ______________________________.



ex. $f(x) = \frac{1}{x}$

We can see that both x and the y-axis are asymptotes of this graph.

Vertical Asymptotes

From the previous graph, we saw that there were two different types of asymptotes at play. There was a ________ asymptotes (the y-axis), as well as a ________________________ asymptote (the x-axis). So for every rational function graph, we must consider both.

Recall that the vertical lines represent the horizontal or the x-coordinates. **Thus, to find vertical asymptotes, we must consider the possible x-coordinates that would make the rational functions undefined, i.e. what x-value makes the ________________________ 0?**

ex. $f(x) = \frac{1}{x}$ For this function, it's obvious that the only place the function is undefined would be when $x = 0$, which is the y-axis. Therefore, it becomes the **vertical asymptotes**.

Examples

Find the vertical asymptotes of the following functions.

1. $f(x) = \frac{x-6}{x+2}$ 2. $g(x) = \frac{8}{2x-9}$ 3. $h(x) = \frac{x-9}{5}$

Horizontal Asymptotes

Horizontal asymptotes are horizontal lines, which represents a certain y-value (y=…). The method for finding horizontal asymptotes is as follows:

Let n be the leading exponent of the numerator and m be the leading exponent of the denominator.

(a). If , i.e. higher degree in the denominator, the horizontal asymptotes is .

(b). If , then the horizontal asymptote is

(c). If , i.e. higher degree in the numerator, then __________________________ exists.

Example

Find the horizontal asymptotes of the following functions.

1. $f(x) = \frac{x^2-6}{x^3+2}$ 2. $g(x) = \frac{8x}{2x-9}$ 3. $h(x) = \frac{9x^4}{5}$

PreCalculus Name: Period: Date:

Slant or Oblique Asymptotes

For situations where no horizontal asymptotes exist, i.e. higher degree in the numerator, there may still exist a ___________________ **or oblique** asymptote. Finding such asymptote is a rather easy process, as it is simply done by ________________ (long division is needed here).

Ex. $f(x) = \frac{x^2-4x-5}{x-3}$

Here we can easily see that no horizontal asymptote exists. We just need to divide to see if there is a slant or oblique asymptote.

Examples

Find the asymptotes of the following functions. If no horizontal asymptote exists, find slant or oblique asymptotes.

1. $f(x) = \frac{5x+21}{x^2+10x+25}$

2. $f(x) = \frac{x^3+3x^2}{x^2-4}$

3. $f(x) = \frac{x^2-3x-4}{2x^2+4x}$

4. $f(x) = \frac{2x^2+7x-4}{x^2+x-2}$

Homework 10/30
TB pg. 313 #11-23 (odd)