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Lesson 3-6: Rational Functions**Objective**

Students will...

- Be able to understand what rational functions are and their behaviors.
- Be able to find the x and the y intercepts of rational functions.

Rational Functions

Whenever we hear the word "rational" in mathematics, it'd be safe to say many of us think of fractions. Hence, a rational function would be most commonly described as a "fractional" function. This is in essence true!

A rational function is a function of the form $r(x) = \frac{P(x)}{Q(x)}$, where P and Q are polynomials. We are also assuming that $P(x)$ and $Q(x)$ have no factor in common, i.e. they are completely _____.

Behaviors of Rational Functions

Rational functions are often given special attention because, while they fit the standard definition of a function (one output for every input), they are quite unique in terms of their _____ and _____.

Consider the following rational function, $f(x) = \frac{1}{x}$

We can already see that there is something we need to make sure of, and that is the fact that $x \neq 0$, since a fraction is not defined when the denominator is a zero.

Also, as x or the denominator _____, the overall function _____, and as x or the denominator _____, the overall function _____.

Ex. $\frac{1}{2} > \frac{1}{12} > \frac{1}{45667}$

So, the behavior of this rational function, $f(x) = \frac{1}{x}$ can be written as,

$$\lim_{x \rightarrow \infty} f(x) = 0 \quad \text{and} \quad \lim_{x \rightarrow 0} f(x) = \infty$$

"The limit _____"

"The limit _____"

X and the Y-Intercepts of Rational Functions

Although we have observed how rational functions behave in a unique way, the concept of finding the x and the y intercepts remain the same for all functions.

Ex. Find the x and the y-intercepts of the function $f(x) = \frac{x-2}{3}$

Y-int:

X-int:

Example

Find the x and the y intercepts of the following rational functions

1. $f(x) = \frac{1}{x}$

2. $r(x) = \frac{x}{2}$

3. $g(x) = \frac{x-5}{x-2}$

4. $f(x) = \frac{x^2-3x-18}{x+4}$

5. $r(x) = \frac{x^2+6}{2}$