PreCalculus Name: Period: Date:

10/27

Lesson 3-5: Complex Zeros I: Fundamental Theorem of Algebra

Objective
Students will…

- Be able to understand what the Fundamental Theorem of Algebra says.
- Be able to factor any polynomial completely using a combination of factoring techniques, synthetic division, and quadratic formula.

Square Root of Negative Numbers
We observed in the past that real numbers alone had some limitations when solving for certain quadratic equations. This was due to the fact that some quadratics required taking the square root of a negative number. For example, to find the zeros of the following polynomial,

$$P(x) = x^2 - x + 1$$

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)} \quad \text{Quadratic Formula}$$

$$x = \frac{1 \pm \sqrt{1-4}}{2} = \frac{\mathbf{1} \pm \sqrt{\mathbf{-3}}}{\mathbf{2}}$$

In the past, we'd simply write, "no ________ solutions," for such equations. So, in order to solve all quadratic equations, mathematicians created an expanded number system called, ___________________________________.

Fundamental Theorem of Algebra
Now that we have eliminated the any limitations to solving quadratic equations by the use of complex numbers, the following theorems result:
Fundamental Theorem of Algebra-

Complete Factorization Theorem-

Factoring a Polynomial Completely
We now use the various techniques that we have acquired to factor polynomials completely, and find all the zeros of any given polynomial.
Ex: Let $P(x) = x^3 - 3x^2 + x - 3$. Find all the zeros and factor completely.

Example

Let $Q(x) = x^3 - 2x + 4$. Find all the zeros and factor completely.

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Zeros Theorem
We have another theorem regarding zeros or solutions of a polynomial.
Zeros Theorem-

Example: $P(x) = (x-1)^3(x+2)^2(x+3)^5$
Zeros: 1 -2 -3
Multiplicity:
So, here we see that $P(x)$ with degree 10 has exactly ______ zeros.

Example

Factor completely, find all zeros, and state the multiplicity of each zero.

$$P(x) = 3x^5 + 24x^3 + 48x$$

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