Warm Up 10/23

Use synthetic division (or long division) to divide $P(x)$ by $D(x)$.

1. $P(x) = 3x^2 + 5x - 4, D(x) = x + 3$

$$\begin{array}{r|rrr} -3 & 3 & 5 & -4 \\ & \downarrow & -9 & 12 \\ \hline & 3 & -4 & 8 \end{array}$$

$3x - 4 + \frac{8}{x+3}$

2. $P(x) = x^4 - x^3 + 4x + 2, D(x) = x^2 + 3$

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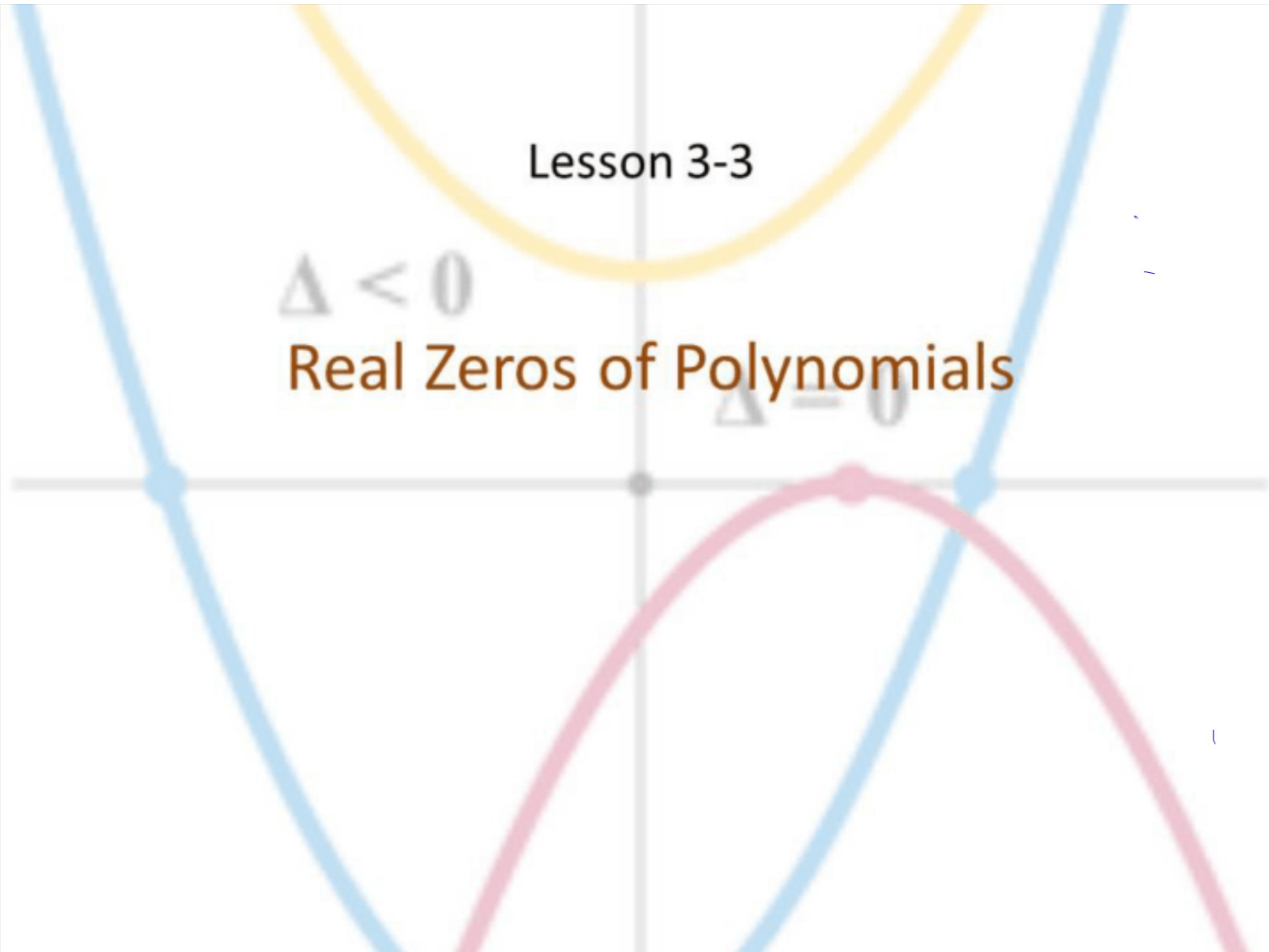
$x^2 + 3 \,\overline{)\, x^4 - x^3 + 0x^2 + 4x + 2}$ quotient $x^2 - x - 3$

$7x + 11$

$x^2 - x - 3 + \frac{7x+11}{x^2+3}$

Lesson 3-3

Real Zeros of Polynomials



Objective

Students will…

- Be able to know how to use synthetic division and factoring to find real zeros of polynomials.
- Be able to apply the Rational Zeros Theorem to find real zeros of polynomials.

Rational Zeros

Consider the following polynomial in two different forms:

$P(x) = (x - 2)(x - 3)(x + 4)$ Factored Form

$P(x) = x^3 - x^2 - 14x + 24$ Expanded Form

From the factored form we can see that zeros of $P(x)$ are 2, 3, and 4. We can also see that all three zeros 2, 3, and 4, are factors of the constant term 24. This result can be generalized.

Rational Zeros Theorem

Rational Zeros Theorem- If the polynomial $P(x)$ integer coefficient, then every rational zero of P is of the form: $\frac{p}{q}$

where p is a factor of the constant coefficient, and q is a factor of the leading coefficient.

Example: For polynomial, $P(x) = x^3 - 3x + 2$

The leading coefficient is 1, and the constant coefficient is 2. So, the zeros of this polynomial must be of the form

$\frac{Factors\ of\ 2}{Factors\ of\ 1} = \frac{\pm 2}{1}, \frac{\pm 1}{1} = \pm 2, \pm 1$ (four possibilities). When tested, we can see that the zeros are 1 and -2.

Example

Factor the polynomial, $\mathrm{P(x)} = 2x^3 + x^2 - 13x + 6$

By Rational Zero Theorem, possible zeros are $\frac{Factor\ of\ 6}{Factor\ of\ 2}$

Factors are 6 are $\pm 1, \pm 2, \pm 3, \pm 6$ and the factors of 2 are $\pm 1, \pm 2$. Thus, the possible rational zeros are

$$\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{6}{1}, \quad \pm\frac{1}{2}, \pm\frac{2}{2}, \pm\frac{3}{2}, \pm\frac{6}{2}$$

Simplifying them, our possibilities are $\pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{3}{2},$ totaling 12 possibilities. So here we must test all possibilities until we find one of the zeros. We can do this in two different ways, one is to plug each into the polynomial, the other is to use synthetic division. But, since either way we must resort to division in order to reduce the polynomial, it's more efficient to use synthetic division.

Example (Cont.)

So, for our polynomial $\mathrm{P(x)} = 2x^3 + x^2 - 13x + 6$, our possible zeros are $\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$. Let's do synthetic division until we find one that gives us no (0) remainder.

1 | 2 1 -13 6
↓ 2 3
2 3 -10

-1 | 2 1 -13 6
-2 1
2 -1 -12

2 | 2 1 -13 6
↓ 4 10 -6
2 5 -3 (0)

$(x-2)(2x^2+5x-3)$

$=(x-2)(2x-1)(x+3)$

2, -3, $\frac{1}{2}$

-6, -1, +6, 5

$(x-\frac{1}{2})(x+\frac{6}{2})$

$(2x-1)(x+3)$

Finding the rational zeros of a polynomial

So, this whole process can be laid out as follows:

1. List all possible zeros. List all possible rational zeros using the Rational Zero Theorem.
2. Divide. Use synthetic division to evaluate the polynomial at each of the possibilities for zeros that you found in step 1. Do this until you end with a remainder 0.
3. Repeat. Repeat steps 1 and 2 for the quotient. Stop when you reach a quotient that is quadratic (degree 2) or anything that can be factored easily. Once the quotient is reduced to a quadratic, you are done because you can either factor or use quadratic formula.

Example

Find all the real zeros of $P(x) = -x^3 - 3x^2 + 13x + 15$

all the possibilities of zero

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$$\frac{\text{Factors } 15}{\text{Factors } -1} = \frac{1}{1}, \frac{3}{1}, \frac{5}{1}, \frac{15}{1} = \pm 1, \pm 3, \pm 5, \pm 15$$

$$\begin{array}{r|rrrr} 1 & -1 & -3 & 13 & 15 \\ & \downarrow & -1 & -4 & 9 \\ \hline & -1 & -4 & 9 & \end{array}$$

$$\begin{array}{r|rrrr} -1 & -1 & -3 & 13 & 15 \\ & \downarrow & 1 & 2 & -15 \\ \hline & -1 & -2 & 15 & 0 \end{array}$$

$(x+1)(-x^2-2x+15)$

$-(x+1)(x^2+2x-15)$

$-(x+1)(x+5)(x-3)=0$

$x = -1, -5, 3$

Example

Find all the real zeros of $P(x) = x^4 - 5x^3 - 5x^2 + 23x + 10$

List the possibilities: $\frac{\text{Factors } 10}{\text{Factors } 1} = \frac{1}{1}, \frac{2}{1}, \frac{5}{1}, \frac{10}{1} = \pm 1, \pm 2, \pm 5, \pm 10$

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-2	1	-5	-5	23	10
	↓	-2	14	-18	-10
	1	-7	9	5	0

$(x+2)(x^3 - 7x^2 + 9x + 5)$ $\pm 5, \pm 1$

$(x+2)(x-5)(x^2-2x-1)$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{2 \pm \sqrt{4 - 4(1)(-1)}}{2(1)} = \frac{2 \pm \sqrt{4+4}}{2}$

$= \frac{2 \pm \sqrt{8}}{2}$

$\frac{2+\sqrt{8}}{2}, \frac{2-\sqrt{8}}{2}$

$x = -2, 5, \frac{2+\sqrt{8}}{2}, \frac{2-\sqrt{8}}{2}$

Homework 10/23

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$2 \rightarrow \frac{10}{5}$

$$\begin{array}{r|l} 2 & 10 \\ & -10 \\ \hline & 0 \end{array}$$

$x^3 - 3x^2 + 3x - 1$

$c = 1$

$$\begin{array}{r|rrrr} 1 & 1 & -3 & 3 & -1 \\ & \downarrow & 1 & -2 & 1 \\ \hline & 1 & -2 & 1 & \boxed{0} \end{array}$$

$x - 1$

$x - c$