Warm Up 10/21

Let $f(x) = \frac{2}{x}$, $g(x) = \frac{4}{x+4}$. Find

1. $f \circ g$

$f(g(x))$

$f\left(\frac{4}{x+4}\right)$

$= \dfrac{2}{\frac{4}{x+4}}$

$= \frac{\cancel{2}}{1} \cdot \frac{x+4}{\cancel{4}\,2} = \boxed{\frac{x+4}{2}}$

2. $g \circ f$

$g(f(x))$

$g\left(\frac{2}{x}\right)$

$= \dfrac{4}{\frac{2}{x} + \frac{4x}{x}}$

$= \dfrac{4}{\frac{2+4x}{x}} = \dfrac{4x}{2+4x}$

$= \dfrac{\cancel{2}4x}{\cancel{2}(1+2x)} = \boxed{\frac{2x}{1+2x}}$

3. $f \circ f$

$f(f(x))$

$f\left(\frac{2}{x}\right)$

$\dfrac{2}{\frac{2}{x}}$

$= \dfrac{\cancel{2}x}{\cancel{2}}$

$= \boxed{x}$

4. $g \circ g$

$g(g(x))$

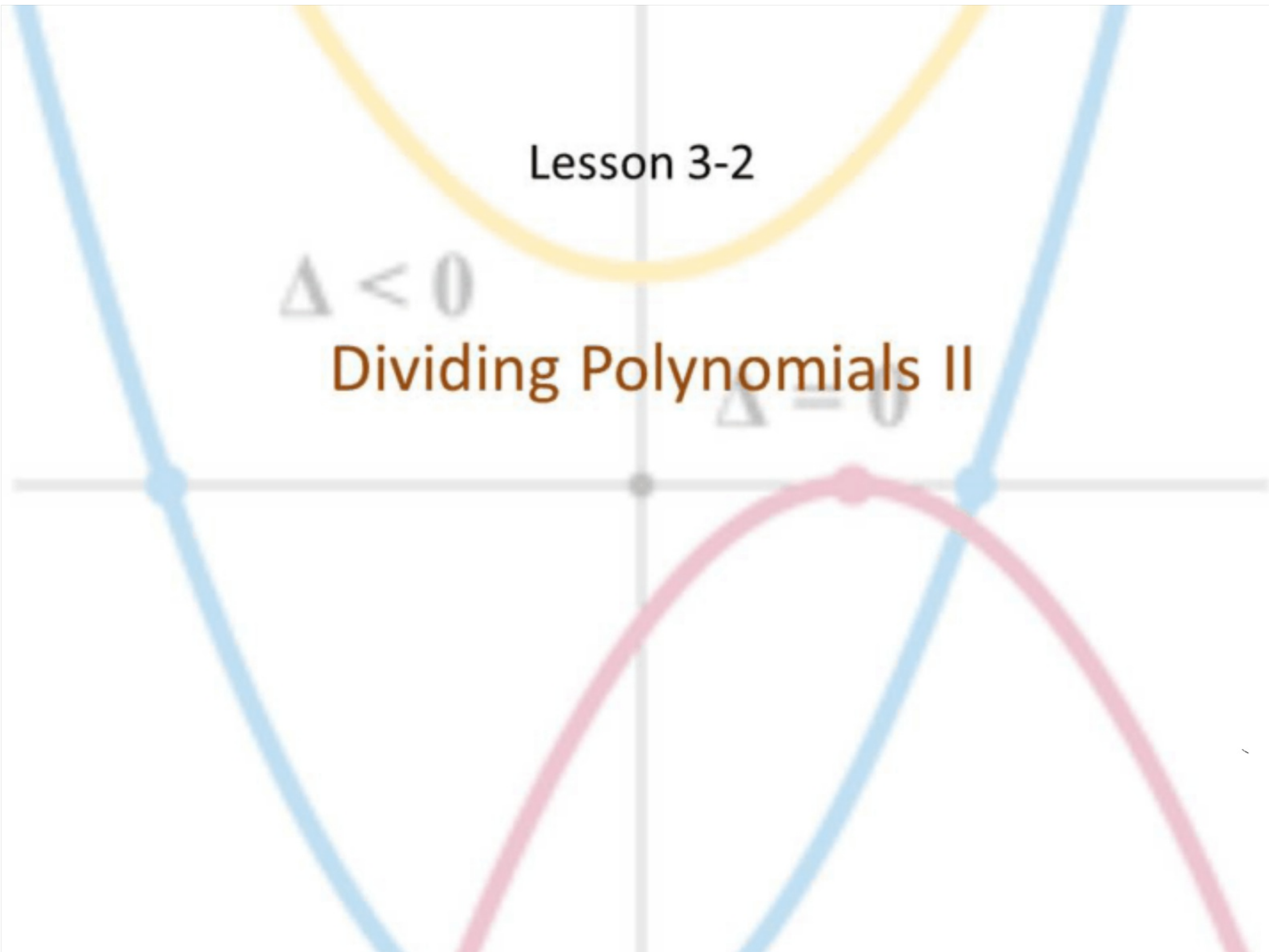
$g\left(\frac{4}{x+4}\right)$

$\dfrac{4}{\frac{4}{x+4} + \frac{4}{1}}$

$\dfrac{\cancel{4}}{\cancel{4}\left(\frac{1}{x+4} + 1\right)} = \boxed{\dfrac{1}{\frac{1}{x+4} + 1}}$

Lesson 3-2

Dividing Polynomials II



Objective

Students will…

- Be able to know how to do synthetic division.
- Be able to relate synthetic division with the remainder theorem to find remainders.
- Be able to use synthetic division and Factor Theorem to identify zeros of a polynomial.

Synthetic Division

$(x+3)(x+1)=0$

Although long division will always get the job done with dividing polynomials, synthetic division is a quicker method. The only drawback to synthetic division is that it can only be used when the divisor is of the form $x - c$. Here are few things to keep in mind when using synthetic division:

- We only need to use the coefficient of each term. x^3+0x^2+x-7
- Need to make sure to include "0" in places where a degree term is missing. For example, for polynomial $x^3 + x - 7$, the coefficient we use would be 1, 0, 1, -7 (0 for the degree 2 term).
- For our divisor $x - c$ the constant we use as our divisor is $-c$. For example, for divisor $x - 8$, the constant we use would be $-(-8) = 8$.

Example

Much like how we first learned how to divide numbers, we can use long division to divide polynomials.

Example: Divide $2x^3 - 7x^2 + 5$ by $x - 3$ using synthetic division.

$$\begin{array}{r|rrrr} 3 & 2 & -7 & 0 & 5 \\ & \downarrow & 6 & -3 & -9 \\ \hline & \boxed{2 \quad -1 \quad -3} & & & -4 \end{array}$$

→ remainder.

$2x^3 - 7x^2 + 5 = (2x^2 - x - 3)(x - 3) - 4$

Q: $2x^2 - x - 3$

Example

Use synthetic division to divide $\mathrm{P(x)} = 5x^3 - 2x^2 + x - 10$ by $x - 3$

$$\begin{array}{r|rrrr} 3 & 5 & -2 & 1 & -10 \\ & \downarrow & & & \\ \hline & 5 & & & \end{array}$$

Q: $5x^2 + 13x + 40$

R: 110

Remainder Theorem

Synthetic division is useful because it can sometimes cut time on evaluating polynomials.

The Remainder Theorem- If polynomial $P(x)$ divided by $x - c$, then the remainder is the value of $P(c)$.

$X - 3$ $\quad$ 3

So, applying to our previous example, we know that 110 (the remainder) is the value of $P(3)$.

$P(c) = $ Remainder

R: 110 $\quad$ $\boxed{P(3) = 110}$

$P(c) = 0$

Example

Let $P(x) = 3x^5 + 5x^4 - 4x^3 + 7x + 3$. Divide by $x + 2$ using synthetic division. Then, use the remainder theorem to evaluate $P(-2)$.

10
5 factor.

Factor Theorem

~~x^2+3x+3~~
x^2+3x+2.

The last theorem to observe in this section is Factor Theorem, which says that zeros of polynomials correspond to factors.

$(x+1)(x+2)$

The Factor Theorem- c is a zero of P if and only if $x - c$ is a factor of $P(x)$.

So, if a certain c is a zero of any polynomial, performing either long division or synthetic division by the divisor $x - c$ should yield **no** remainder (or remainder 0).

$$\begin{array}{r|rrr} -1 & 1 & 3 & 2 \\ & & -1 & -2 \\ \hline & 1 & 2 & \boxed{0} \end{array}$$

$\frac{4}{3} \times \frac{3}{1} = 4$

Example

Use the Factor Theorem to show that $x - c$ is a factor of $P(x) = x^3 - 3x^2 + 3x - 1$, $c = 1$ and factor completely to find all the zeros.

$P(1) = 0$

$$\begin{array}{r|rrrr} 1 & 1 & -3 & 3 & -1 \\ & \downarrow & 1 & -2 & 1 \\ \hline & 1 & -2 & 1 & \boxed{0} \end{array}$$

$\boxed{x-1}$

$(x-1)(x^2-2x+1) = P(x)$ $\quad \boxed{x=1}$

$(x-1)(x-1)(x-1) = 0$

Homework 10/22

TB pg. 271 #35-55 (e.o.o)