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Lesson 3-2: Dividing Polynomials**Objective**

Students will...

- Be able to use long division and synthetic division to divide a polynomial.
- Be able to identify the dividend, divisor, quotient, and the remainder after dividing a polynomial using long division.
- Be able to know and use the Remainder and Factor Theorem.

**End Behavior**

Polynomials can also be studied extensively using algebra. From our experience with graphing, we realized that factoring is a powerful tool when it comes to studying polynomials. Recall that factoring really is \_\_\_\_\_ . Hence, to factor, we must learn how to divide polynomials. Dividing polynomials is much like dividing numbers.

Ex.  $\frac{38}{7} = 5 + \frac{3}{7}$

When we divide 38 by 7, we end up with a \_\_\_\_\_ of 5 and the \_\_\_\_\_ of  $\frac{3}{7}$ .

**Long Division**

Much like how we first learned how to divide numbers, we can use long division to divide polynomials.

Example: Divide  $6x^2 - 26x + 12$  by  $x - 4$ .

So, we are done when the long division ends with a polynomial that is of \_\_\_\_\_ degree than what we divided by. In our case, we divided by  $x - 4$  (degree 1) and ended up with 4 (degree 0). So, we can interpret our result in two ways:

$$\frac{6x^2 - 26x + 12}{x - 4} =$$

Or

$$6x^2 - 26x + 12 =$$

Example

For the following, find the quotient and remainder using long division.

1.  $\frac{x^2 - 6x - 8}{x - 4}$

2.  $\frac{x^3 + 3x^2 + 4x + 3}{3x + 6}$

3.  $\frac{2x^5 - 7x^4 - 13}{4x^2 - 6x + 8}$

**Synthetic Division**

Although long division will always get the job done with dividing polynomials, \_\_\_\_\_ division is a quicker method. The only drawback to synthetic division is that it can only be used when the divisor is of the form \_\_\_\_\_ . Here are few things to keep in mind when using synthetic division:

- We only need to use the \_\_\_\_\_ of each term.
- Need to make sure to include "\_\_\_\_\_" in places where a degree term is missing. For example, for polynomial  $x^3 + x - 7$ , the coefficient we use would be 1, 0, 1, -7 (0 for the degree 2 term).
- For our divisor  $x - c$  the constant we use as our divisor is  $-c$ . For example, for divisor  $x - 8$ , the constant we use would be  $-(-8) = 8$ .

## Example

Divide  $2x^3 - 7x^2 + 5$  by  $x - 3$  using synthetic division.

Use synthetic division to divide  $P(x) = 5x^3 - 2x^2 + x - 10$  by  $x - 3$

**Remainder Theorem**

Synthetic division is useful because it can sometimes cut time on evaluating polynomials.

The Remainder Theorem- If polynomial  $P(x)$  divided by  $x - c$ , then the remainder is the value of \_\_\_\_\_.  
So, applying to our previous example, we know that 110 (the remainder) is the value of  $P(3)$ .

## Example

Let  $P(x) = 3x^5 + 5x^4 - 4x^3 + 7x + 3$ . Divide by  $x + 2$  using synthetic division. Then, use the remainder theorem to evaluate  $P(-2)$ .

**Factor Theorem**

The last theorem to observe in this section is \_\_\_\_\_ Theorem, which says that zeros of polynomials correspond to factors.

The Factor Theorem-  $c$  is a zero of  $P$  if and only if \_\_\_\_\_ is a factor of  $P(x)$ .  
So, if a certain  $c$  is a zero of any polynomial, performing either long division or synthetic division by the divisor  $x - c$  should yield \_\_\_\_\_ remainder (or remainder 0).

## Example

Use the Factor Theorem to show that  $x - c$  is a factor of  
 $P(x) = x^3 - 3x^2 + 3x - 1$ ,  $c = 1$  and factor completely to find all the zeros.