

Warm Up 10/14

For the following functions, find the zeros and describe their end behavior.

1. $P(x) = x^2 - 4$ *even +*

$x = \pm 2$

$x \rightarrow \infty, y \rightarrow \infty$
 $x \rightarrow -\infty, y \rightarrow \infty$

3. $R(x) = -(x-2)^2(x+1)^3$

$x = 2, -1$ *odd neg*

$x \rightarrow \infty, y \rightarrow -\infty$
 $x \rightarrow -\infty, y \rightarrow \infty$

2. $Q(x) = -2(x+2)^4$ *even -*

$x = -2$

$x \rightarrow \infty, y \rightarrow -\infty$
 $x \rightarrow -\infty, y \rightarrow -\infty$

4. $S(x) = x^3 + 3x^2 - 9x - 27$ *odd +*

$0 = x^2(x+3) - 9(x+3)$

$0 = (x^2 - 9)(x+3) \Rightarrow x = \pm 3$

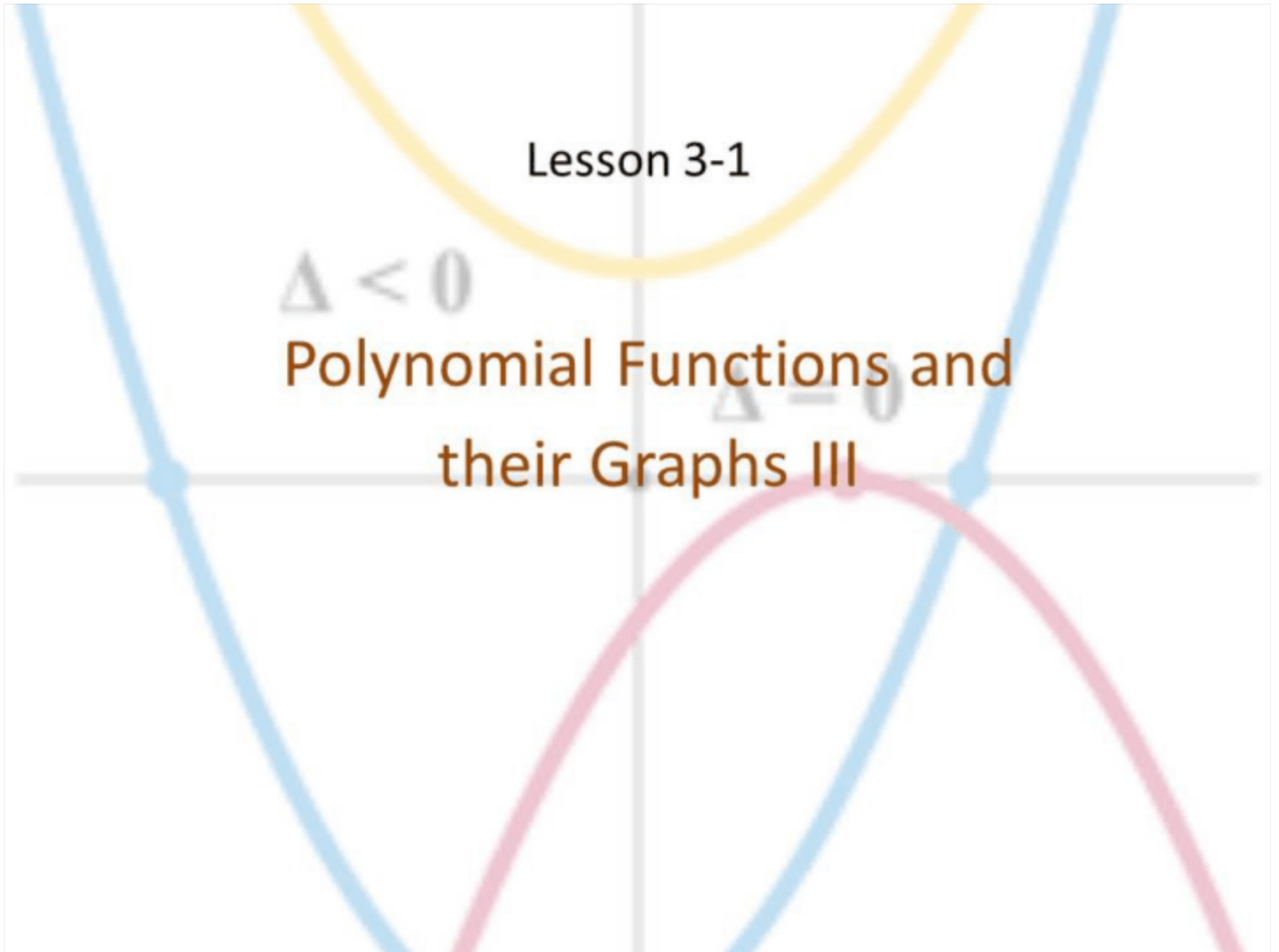
$x \rightarrow \infty, y \rightarrow \infty$
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Lesson 3-1

$\Delta < 0$

Polynomial Functions and
their Graphs III

$\Delta = 0$



Objective

Students will...

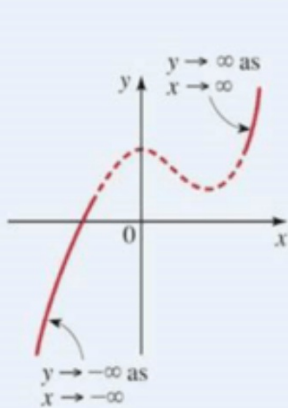
- Be able to find and apply the multiplicity of each zero to graph polynomial functions.

End Behavior

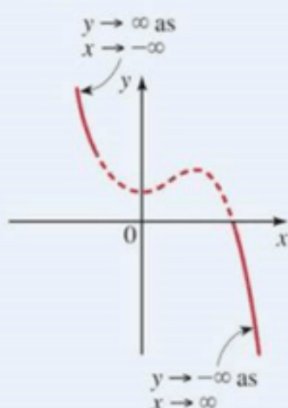
END BEHAVIOR OF POLYNOMIALS

The end behavior of the polynomial $P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ is determined by the degree n and the sign of the leading coefficient a_n , as indicated in the following graphs.

P has odd degree

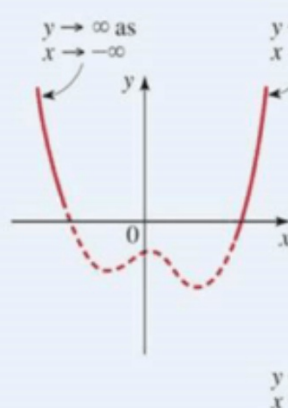


Leading coefficient positive

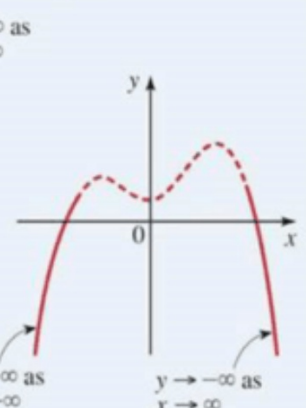


Leading coefficient negative

P has even degree



Leading coefficient positive



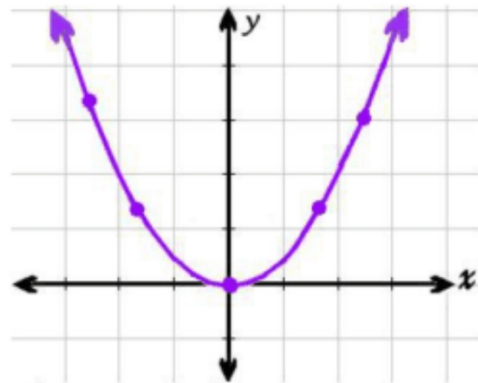
Leading coefficient negative

Shape of the Graph Near a Zero

As we can observe from various graphs, we see that some **cross** the x-axis, while some do not. For example,



This cubic graph crosses the x-axis 3 times.

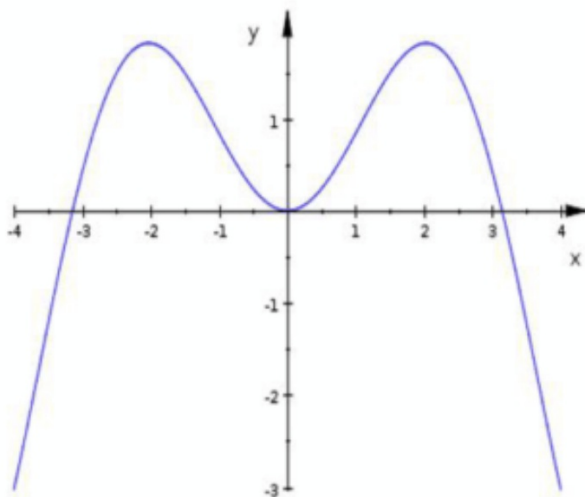


This parabola never crosses the x-axis.

Note: Touching and crossing are different things!

Shape of the Graph Near a Zero

In fact, some graphs contain a mixture of crossing and no crossing behavior. Consider,



Here, we can see that while the graph crosses the x-axis at the two ends, it does not cross the x-axis in the middle. (Note: the graph is still **touching** the x-axis in all places, so they are still considered as zeros of the graph).

Crossing Behavior

We can also observe a nice pattern regarding the x-axis crossing behaviors of polynomial graphs. The pattern has to do with the multiplicity of every zero, i.e. the exponent attached to them.

Example: $P(x) = x^4(x - 2)^3(x + 1)^2$

$$P(x) = x x x x (x - 2)(x - 2)(x - 2) (x + 1)(x + 1).$$

For the above polynomial function $P(x)$, its zeros are 0, 2, and -1. The multiplicity of these zeros are the exponents that are attached to each of them. So, the x-intercept 0 has the multiplicity of 4, while 2 has the multiplicity of 3, and -1 has the multiplicity of 2.

Multiplicity and the Crossing Behavior

With that said, the pattern regarding the x-axis crossing behavior is as follows

For every odd multiplicity, the graph at that particular x-intercept, will **cross** the x-axis.

For every even multiplicity, the graph at that particular x-intercept, will **not cross** the x-axis.

So from our previous example, since the x-intercepts 0 and -1 had an even multiplicity, the graph will not cross the x-axis at those points. In contrast, at the intercept 2, the graph will cross the x-axis because it had an odd multiplicity.

Graphing Polynomials

How is this useful? Well, understanding where the graph does and doesn't cross the x-axis will seriously aid in graphing the polynomials.

Example: Graph the polynomial, $P(x) = x^4(x - 2)^3(x + 1)^2$

Zeros: $(2) (4) (3)$
 $-1, 0, 2$

Y-intercept: $P(0) = 0$

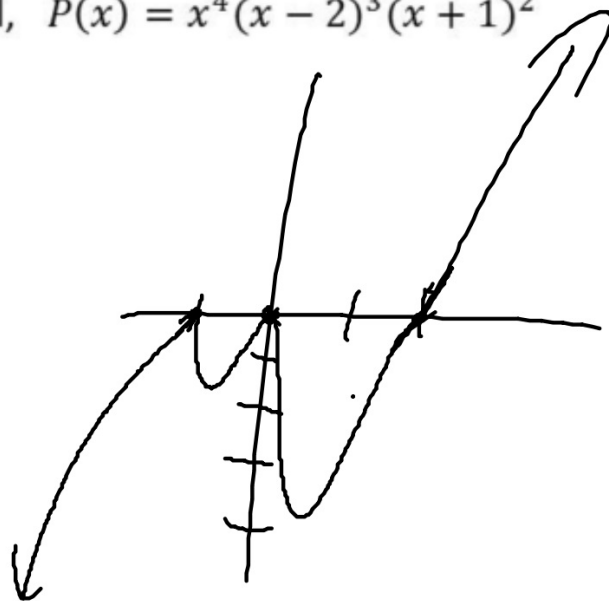
Degree: 9 (odd)

+ or - ? : +

End Behavior:

$x \rightarrow \infty, y \rightarrow \infty$

$x \rightarrow -\infty, y \rightarrow -\infty$



Example

Graph the polynomial: $R(x) = (x - 1)^2(x - 2)(x - 5)^3$

Zeros: $\overset{(2)}{1}, \overset{(1)}{2}, \overset{(3)}{5}$

Y-intercept: $R(0) = 1 \cdot 2 \cdot 125 = 250$

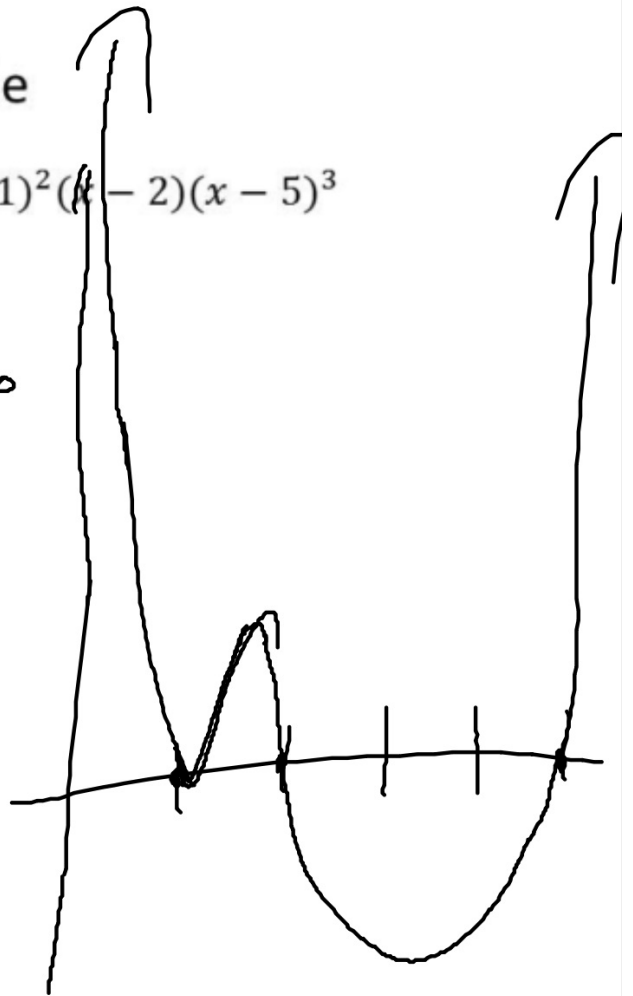
Degree: 6 (even)

+ or -?: +

End Behavior:

$x \rightarrow \infty, y \rightarrow \infty$

$x \rightarrow -\infty, y \rightarrow \infty$



Example

Graph the polynomial: $T(x) = -x^4 + 3x^3 - 2x^2$

$$0 = -x^2(x^2 - 3x + 2) = -x^2(x-2)(x-1)$$

Zeros: 0, 1, 2

Y-intercept: $T(0) = 0$

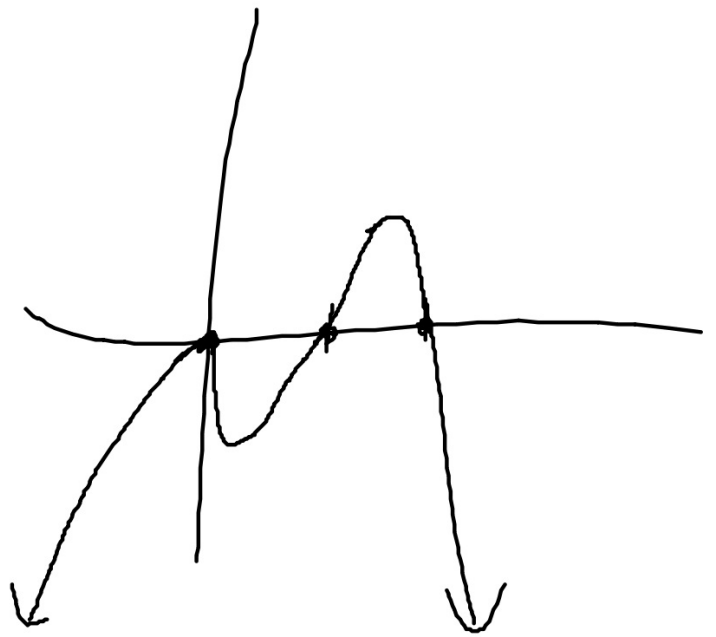
Degree: 4 (even)

+ or - ? : -

End Behavior:

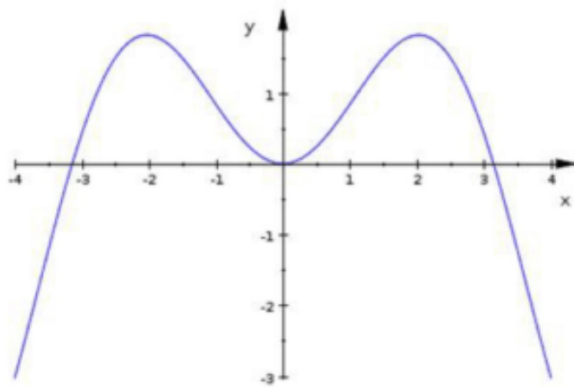
$$x \rightarrow \infty, y \rightarrow -\infty$$

$$x \rightarrow -\infty, y \rightarrow -\infty$$



Local Extrema (Maxima and Minima)

The last thing to observe in this section is the local extrema. Extrema refers to both maxima and minima of a graph. For Example,



We can see that this graph contains three local extrema.

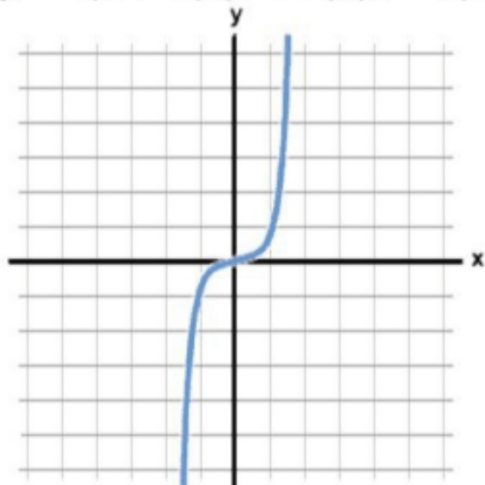
Local Extrema Principle

The Local Extrema Principle states that for every polynomial of degree, n , the graph has **at most** $n - 1$ local extrema.

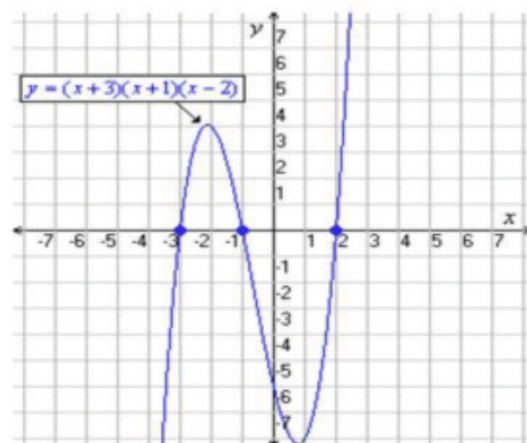
So for example, $T(x) = -x^4 + 3x^3 - 2x^2$, can have no more than $4 - 1 = 3$ local extrema. We can see this on our graph.

Also, $P(x) = x^4(x - 2)^3(x + 1)^2$, can have no more than $9 - 1 = 8$ local extrema. We can also see this on our graph.

The key word here is of course, “**at most.**” Hence, a polynomial can have less than $n - 1$ local extrema. In fact, standard function $P(x) = x^3$ does not have any local extrema, while $y = (x + 3)(x + 1)(x - 2)$ has exactly 2 (which is okay since $3-1=2$).



$$P(x) = x^3$$



$$y = (x + 3)(x + 1)(x - 2)$$

Homework 10/14

TB pg. 262 #11-35 (e.o.o)

Use the zeros, end behaviors, and the multiplicity to sketch the graph.