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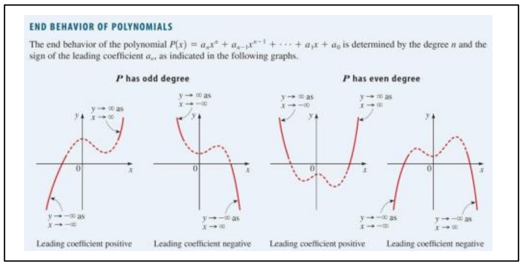
<u>Lesson 3-1c: Polynomial Functions and their Graphs II</u>

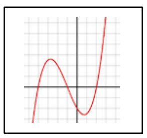
Objective
Students will…
- Be able to find and apply the multiplicity of each zero to graph polynomial functions.

End Behavior

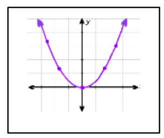


Shape of the Graph Near a Zero
As we can observe from various graphs, we see that some ___________ the x-axis, while some do not. For example,

This cubic graph crosses the x-axis 3 times.

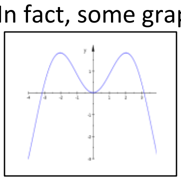


This parabola never crosses the x-axis.



Note: Touching and crossing are different things!

In fact, some graphs contain a ______________________ of crossing and no crossing behavior. Consider,



Here, we can see that while the graph crosses the x-axis at the two ends, it does not cross the x-axis in the middle. (Note: the graph is still _______________ the x-axis in all places, so they are still considered as zeros of the graph).

Crossing Behavior
We can also observe a nice pattern regarding the x-axis crossing behaviors of polynomial graphs. The pattern has to do with the _______________________________of every zero, i.e. the exponent attached to them.

Example: $P(x) = x^4(x-2)^3(x+1)^2$

For the above polynomial function $P(x)$, its zeros are ______________________. The multiplicity of these zeros is the exponent that is attached to each of them. So, the x-intercept 0 has the multiplicity of ____, while 2 has the multiplicity of _____, and -1 has the multiplicity of ____.

Multiplicity and the Crossing Behavior
With that said, the pattern regarding the x-axis crossing behavior is as follows,

For every _________<u>multiplicity</u>, the graph at that particular x-intercept, will **cross** the x-axis.
For every ____________<u>multiplicity</u>, the graph at that particular x-intercept, will **not cross** the x-axis.

So from our previous example, since the x-intercepts 0 and -1 had an even multiplicity, the graph will not cross the x-axis at those points. In contrast, at the intercept 2, the graph will cross the x-axis because it had an odd multiplicity.

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Graphing Polynomials

How is this useful? Well, understanding where the graph does and doesn't cross the x-axis will seriously aid in graphing the polynomials.

Example: Graph the polynomial, $P(x) = x^4(x-2)^3(x+1)^2$

Zeros:
Y-intercept:
Degree:
+ or - ?:
End Behavior:

Graph the polynomial: $R(x) = (x-1)^2(x-2)(x-5)^3$

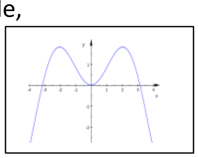
Zeros:
Y-intercept:
Degree:
+ or - ?:
End Behavior:

Graph the polynomial: $\mathrm{T}(x) = -x^4 + 3x^3 - 2x^2$

Zeros:
Y-intercept:
Degree:
+ or - ?:
End Behavior:

Local Extrema (Maxima and Minima)

The last thing to observe in this section is the ______________________________. Extrema refers to both maxima and minima of a graph. For Example,



We can see that this graph contains _________ local extrema.

Local Extrema Principle

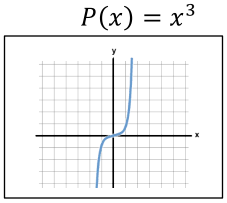
The ____________________________________ states that for every polynomial of degree, *n*, the graph has **at most** $n-1$ local extrema.

So for example, $\mathrm{T}(x) = -x^4 + 3x^3 - 2x^2$, can have no more than local extrema. We can see this on our graph.

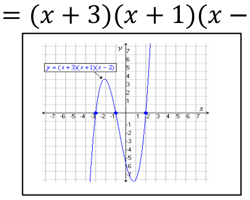
Also, $P(x) = x^4(x-2)^3(x+1)^2$, can have no more than local extrema. We can also see this on our graph.

The key word here is of course, "**at most.**" Hence, a polynomial can have less than $n-1$ local extrema. In fact, standard function $P(x) = x^3$ does not have any local extrema, while $y = (x+3)(x+1)(x-2)$ has exactly 2 (which is okay since 3-1=2).

$P(x) = x^3$



$y = (x+3)(x+1)(x-2)$



Homework 10/14

TB pg. 262 #11-35 (e.o.o) Use the zeros, end behaviors, and the multiplicity to sketch the graph.