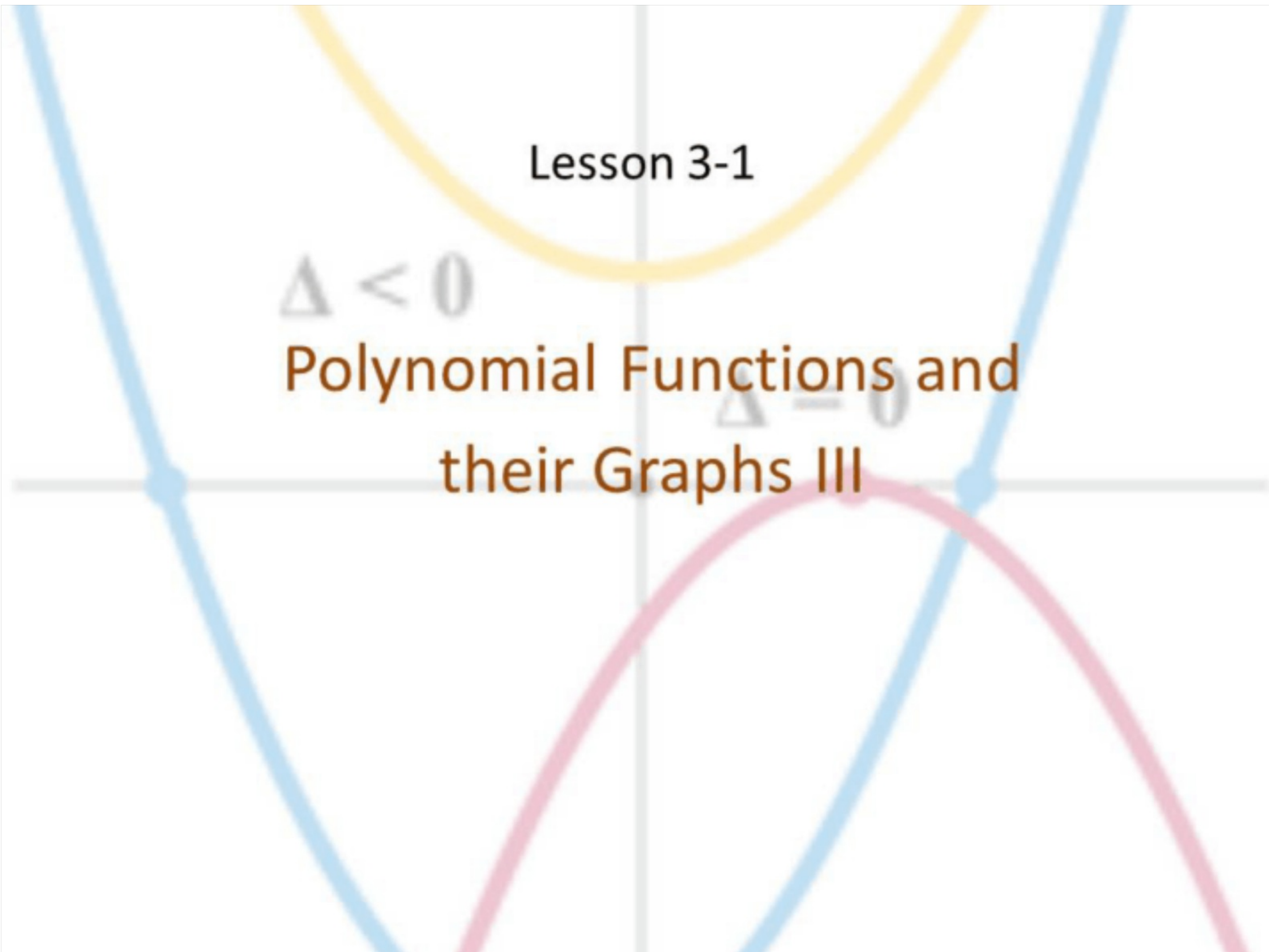
Lesson 3-1

Polynomial Functions and their Graphs III



Warm Up 10/16

Find the zeros of the following polynomial functions.

1. $P(x) = (x-1)(x+1)(x-2)$

$\boxed{x = 1, -1, 2}$

2. $Q(x) = -2x^3 - x^2 + x$

$-x(2x^2 + x - 1) = 0$

$-x(x+\frac{2}{2})(x-\frac{1}{2}) = 0$

$-x(x+1)(2x-1) = 0 \quad x = -1, 0, \frac{1}{2}$

3. $S(x) = x^5 - 9x^3$

$x^3(x^2-9) = 0$

$\sqrt[3]{x^3} = 0 \Rightarrow x = 0$

$x^2 - 9 = 0$

$\sqrt{x^2} = \sqrt{9}$

$x = \pm 3$

4. $R(x) = \frac{1}{5}x(x-5)^2$

$\frac{1}{5}$

$0 = x(x-5)^2$

$x = 0$

$\sqrt{(x-5)^2} = \sqrt{0}$

$x - 5 = 0$

$x = 5$

5. $P(x) = (x^3 + x^2)(-x - 1)$

$x^2(x+1) - 1(x+1)$

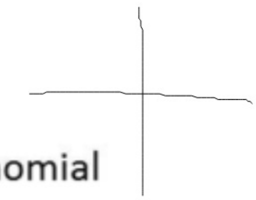
$(x^2 - 1)(x+1) = 0$

$x^2 - 1 = 0$

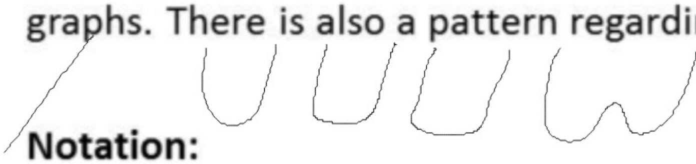
$\sqrt{x^2} = \sqrt{1}$

$x = \pm 1$

End Behavior



We saw a few distinct natures and patterns involving polynomial graphs. There is also a pattern regarding their <u>end behavior</u>.



Notation:

$\boxed{x \to \infty}$ means "*x* becomes large in the positive direction"

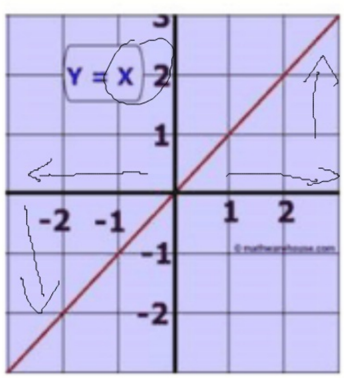
$\boxed{x \to -\infty}$ means "*x* becomes large in the negative direction"

$\boxed{y \to \infty}$ means "*y* becomes large in the positive direction"

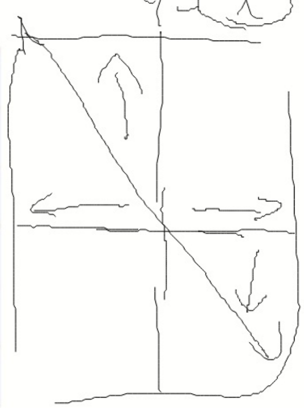
$\boxed{y \to -\infty}$ means "*y* becomes large in the negative direction"

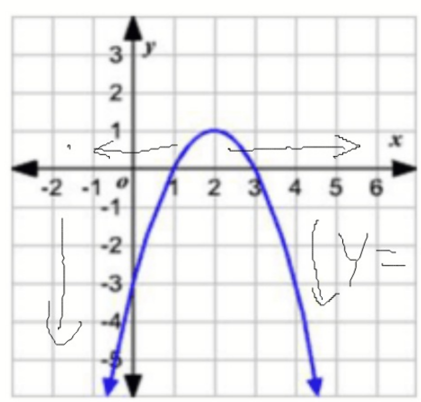
End Behavior

For example, consider the following graph.



y = -x





$y = -x^2$

For $y = x$, $y \to \infty$ as $x \to \infty$,
and $y \to -\infty$ as $x \to -\infty$

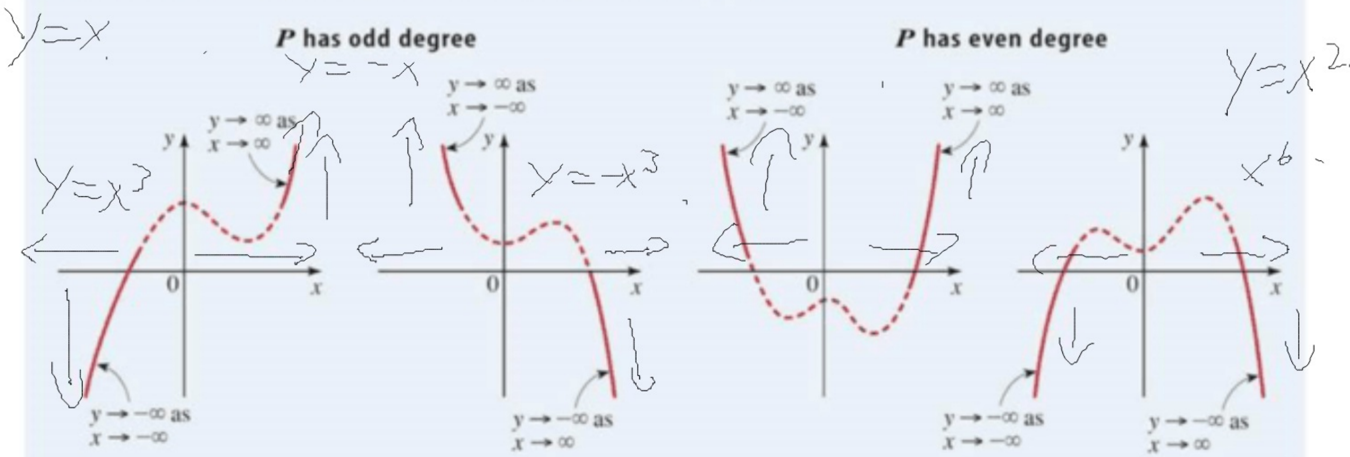
For $y = -x^2$, $y \to -\infty$, as $x \to \infty$,
as well as $x \to -\infty$

End Behavior

So, we can conclude that polynomials take the following patterns.

END BEHAVIOR OF POLYNOMIALS

The end behavior of the polynomial $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ is determined by the degree n and the sign of the leading coefficient a_n, as indicated in the following graphs.

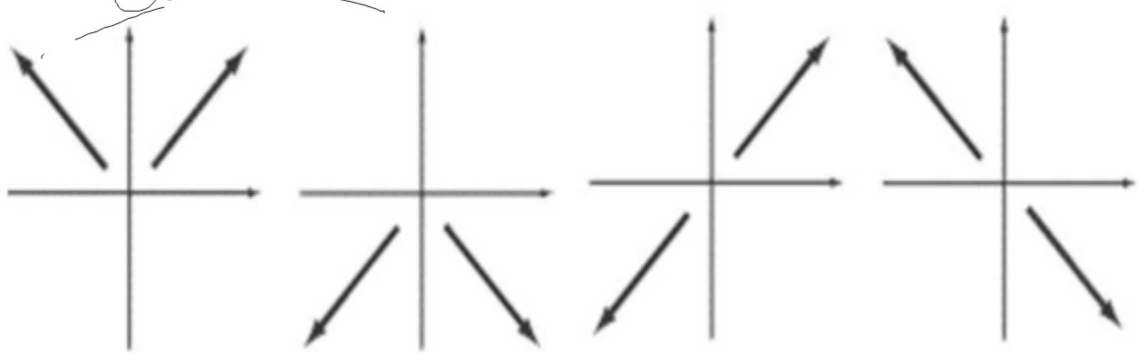


Leading coefficient positive | Leading coefficient negative | Leading coefficient positive | Leading coefficient negative

End Behavior

So, we can conclude that polynomials take the following patterns.

evens Odds



n is even	n is even	n is odd	n is odd
$a_n > 0$	$a_n < 0$	$a_n > 0$	$a_n < 0$

$(x^2)^2 = x^{2\cdot 2}$

Examples

$x^3 \cdot y^3$ $\quad (x^3+2)^2$

xy^{3+3} $\quad (x^3+2)(x^3+2)$

Describe the end behavior of the following functions.

x^2 $\quad$ x^3 $\quad$ Odd Cubic

1. $P(x) = (x-1)(x+2)$

$x \to \infty,\ y \to \infty$

$x \to -\infty,\ y \to \infty$

2. $Q(x) = (x-3)(x+2)(3x-2)$

$x \to \infty,\ y \to \infty$

$x \to -\infty,\ y \to -\infty$

3. $S(x) = \frac{1}{8}(2x^5+3x^3)^2$

x^{10} even +

$x \to \infty,\ y \to \infty$

$x \to -\infty,\ y \to \infty$

4. $R(x) = x^3(x+2)(x-3)^2$

x^6

5. $Q(x) = x^3 + 3x^2 - 4x - 12$

6. $R(x) = \frac{1}{4}(x+1)^3(x-3)$

Homework 10/15/13

TB pg. 262 #11-35 (e.o.o)

Describe the end behavior of each.